1. INTRODUCTION

The polarimetric calibration method and polarimetric decompostition technique using a particle swarm optimization (PSO) are discussed in this report. Recently the polarimetric data analysis techniques are being developed for terrain classification, forest biomass and soil moisture estimations, etc. Thus, polarimetric calibration becomes an important issue for accurate polarimetric analysis[1]-[3]. I propose new polarimetric calibration method based on a particle swarm optimization. This proposed calibration method can estimate cross-talks, channel imbalances and Faraday rotation angle using one trihedral corner reflector and the measured polarimetric SAR data with non-reflection symmetry property. Next, this report deals with a polarimetric decomposition [5][6] considering azimuth rotation. Polarimetric data in urban and mountain areas are affected by azimuth rotation. Thus, the decomposition results derived by same targets with different azimuth rotation angle are different. Some researchers examined to compensate this rotation by mathematical inverse azimuth rotation.

I examine that the differences of covariance matrix before and after the compensation are used to estimate the contributions of three decomposition components. In the following sections, proposed polarimetric calibration and decomposition techniques are explained and some numerical results are discussed.

2. POLARIMETRIC CALIBRATION BASED ON PARTICLE SWARM OPTIMIZATION

2.1 Polarimetric calibration

For a spaceborne SAR system using linear horizontal (H) and vertical (V) polarizations, the polarimetric calibration model can be written as

\[
M = RFSFT + n
\]

where \( M \) and \( RFSFT \) are the measured and synthesized scattering matrices, \( n \) is the noise matrix. A procedure for calibration of polarimetric SAR data that takes into account Faraday rotation is to estimate all parameters in \( R, T \) and \( F \). These parameters are composed of six complex values and one real value. If \( R, T \) and \( F \) are known and \( n \) is ignored, the true scattering matrix can be obtained as

\[
S = F^T R^{-1} M^T F^{-1}
\]

2.2 Polarimetric calibration method using particle swarm optimization

Particle swarm optimization technique can be used to find an approximated global optimal solution to an optimization problem [4] and has been shown to be useful for optimization about a multidimensional problem in various applications. A swarm is modeled by particles in multidimensional search space. These particles have a position and a velocity and move in the search space due to two essential reasoning capabilities which are related to
their own best position and the best position in the swarm. Particles can communicate best positions to each other and adjust their own position and velocity based on these good positions. The velocity \(v\) and position \(x\) of \(n\)th particle are defined as:

\[
v_{k+1} = v_k + C_1 r_1 (p_k - x_k) + C_2 r_2 (g_k - x_k) \quad (5a)
\]

\[
x_{k+1} = x_k + v_{k+1} \quad (5b)
\]

where \(\omega\) is an inertial weight, \(C_1\) and \(C_2\) are an acceleration coefficient, \(r_1\) and \(r_2\) are a random variable. \(p\) is the best position in each particle and \(g\) is the best position in the swarm. \(k\) is an iteration index. In the optimization, the position and velocity of each particle is adjusted to minimize or maximize a fitness of objective function. A procedure of PSO is as follows. At first, this technique prepares a set (swarm) of candidate solutions (particles). Next, the fitness of each particle is evaluated. If a fitness level is reached to a termination condition, it is considered that an approximated solution is found. If the fitness level is not reached to the condition, the velocity and position updates are carried out. In the case of polarimetric calibration, a particle is expressed by two channel imbalance, four cross-talks and Faraday rotation angle.

The objective function of polarimetric calibration is defined as the following. The true polarimetric SAR data \(S\) is expressed as eq. (1) and consists of four polarimetric coefficients with respect to HH, HV, VH and VV. The covariance matrix \(C\) can be obtained from these polarimetric coefficients.

\[
k=[S_{HH} \quad S_{HV} \quad S_{VH} \quad S_{VV}]^T \quad (6)
\]

\[
C=kk^T
\]

\[
\begin{align*}
\{S_{uu}S_{uv}\} & \quad \{S_{uu}S_{uv}\} & \quad \{S_{uv}S_{uv}\} & \quad \{S_{uv}S_{uv}\} \\
\{S_{uv}S_{uv}\} & \quad \{S_{uv}S_{uv}\} & \quad \{S_{uv}S_{uv}\} & \quad \{S_{uv}S_{uv}\} \\
\{S_{uv}S_{uv}\} & \quad \{S_{uv}S_{uv}\} & \quad \{S_{uv}S_{uv}\} & \quad \{S_{uv}S_{uv}\} \\
\{S_{uv}S_{uv}\} & \quad \{S_{uv}S_{uv}\} & \quad \{S_{uv}S_{uv}\} & \quad \{S_{uv}S_{uv}\}
\end{align*}
\]

where \(k\) is a target vector and \(\bar{f}\) is a complex conjugate transpose. When a monostatic radar system is used to observe the terrains on earth, the scattering matrix needs to satisfy scattering reciprocity meaning that \(S_{HH} = S_{VV}\) and \(S_{HV} = S_{VH}\). Thus, I can get some equations from (7).

\[
<S_{HH}S_{HH}>=<S_{VH}S_{VH}>,
\]

\[
<S_{HV}S_{VV}>=<S_{VH}S_{VH}>,
\]

\[
\text{Im}(<S_{HH}S_{VV}>)=0 \quad (8a)-(8c)
\]

Equations (8a)-(8c) have to be satisfied anywhere in polarimetric SAR data. These equations are applied to the objective function of PSO to find the polarimetric calibration parameters. Therefore, it is not needed to use the reflection symmetry condition and the assumption that cross-talks are negligible. However, more equations are needed to obtain six complex and one real unknowns. In order to overcome this problem, a trihedral corner reflector information is used. A scattering matrix of the trihedral corner reflector is written as

\[
S_{mi} = \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}.
\]

From equation (9), more equations are obtained.

\[
<S_{HH}S_{HH}>=<S_{VH}S_{VH}>,
\]

\[
<S_{HV}S_{VV}>=<S_{VH}S_{VH}>,
\]

\[
\text{Im}(<S_{HH}S_{VV}>)=0 \quad (10a)-(10c)
\]

Moreover, in built-up areas, radar polarization orientation shifts occur [7] and a phase difference between like and cross polarizations is changed with wall orientation angle as

\[-90^\circ<\phi<0^\circ : \text{Arg}(S_{HH}S_{HH})=0^\circ \\
0^\circ<\phi<90^\circ : \text{Arg}(S_{HH}S_{HH})=180^\circ \quad (11a,b)
\]

where \(\phi\) is wall orientation angle as shown in Fig.1. Thus, the objective function \(F\) to evaluate the fitness based on reciprocity, trihedral corner reflector and radar polarization shifts is defined by eq.(8), (10) and (11). A particle \(D_n\) is expressed as

\[
D_n = \left[ f_{1,n} \quad f_{2,n} \quad \delta_{1,n} \quad \delta_{2,n} \quad \delta_{3,n} \quad \delta_{4,n} \quad \Omega_n \right] \quad (12)
\]

A scattering matrix calibrated by \(D_n\) is represented as

\[
S(D_n)=F(D_n)^{-1}R(D_n)^{-1}MT(D_n)F(D_n)^{-1} \quad (13)
\]

If an area where trihedral corner reflector does not exist is selected to estimate the fitness, the objective function \(F_{wcr}(D_n)\) can be expressed as

\[
F_{wcr}(D_n) = \frac{\{S_{HH}(D_n)S_{HH}(D_n)\} - \{S_{HH}(D_n)S_{HH}(D_n)\}}{\{M_{HH}\}M_{HH}}
\]

If an area where CR exists is selected, objective function \(F_{cr}(D_n)\) can be written as

\[
F_{cr}(D_n) = \frac{\{S_{HH}(D_n)S_{HH}(D_n)\} - \{S_{HH}(D_n)S_{HH}(D_n)\}}{\{M_{HH}\}M_{HH}}
\]

In addition to these functions, other objective function based on eq.(11) is derived as

\[
F_{wbiuf}(D_n) = \frac{\text{Arg}\{S_{HH}(D_n)S_{HH}(D_n)\} - \text{theoretical \_angle}}{\text{Arg}\{M_{HH}\}M_{HH}}
\]

where theoretical angle (0° or 180°) is determined under the condition that \(\phi\) is known in estimated built-up area. In order to get average polarimetric calibration parameters, I use several estimation areas to calculate objective functions of eq.(14), (15) and (16). Then final objective function becomes as follows.
where $T$ and $L$ are numbers of non CR area and CR area. A flow chart of PSO is shown in Fig.2. In order to restrict a range of search space and to reduce local minima, the calculation scheme of PSO becomes as follows.

Step 1) Estimate $f_1$, $f_2$ and $\Omega$ roughly by PSO. At this time, cross-talks $\delta_1$, $\delta_2$ are assumed to be zero and the search ranges of initial particle with respect to $f_1$, $f_2$ and $\Omega$ is set widely.

Step 2) Make new initial particle set with respect to $f_1$, $f_2$, $\Omega$ and $\delta_1$, $\delta_2$ on the restricted ranges based on results of Step 1 and make the parameters with respect to $\delta_1$, $\delta_2$ based on priori information that cross-talks are smaller than 1.

Step 3) Estimate $f_1$, $f_2$, $\delta_1$, $\delta_2$ and $\Omega$ by PSO using the initial particle set of Step 2).

PSO calculation is terminated when one particle gets a best fitness or after a certain fixed number iteration is reached.

![Fig. 1 Orientation angle of building wall](image)

**Parameters ($f_1$, $\delta_1$, $\delta_2$, $\Omega$)
Fitness function: $\text{Fit}(f_1, \delta_1, \delta_2, \Omega)$**

- Initialize a position of each particle
- Evaluate fitness $\text{Fit}(D_0) \leq \text{Condition}$
- No
- Velocity update
- Position update
- Yes
- End

![Fig. 2 Flow chart of PSO for polarimetric calibration.](image)

3. POLARIMETRIC DECOMPOSITION BASED ON PARTICLE SWARM OPTIMIZATION

3.1. Three-component decomposition

The scattering matrix and covariance matrix are defined as eq.(6) and (7). Due to the backscattering, it is assumed that $S_{HV}$ is equal to $S_{VH}$. Freeman and Durden proposed three-component scattering model for POLSAR image decomposition based on a measured covariance matrix [5][6]. In case of their decomposition, the covariance matrix is decomposed as

$$
\begin{bmatrix}
\mathbf{C}(HV)\
\mathbf{C}(HV)^*\
\end{bmatrix} = f_d \mathbf{C}_{\text{surf}} + f_s \mathbf{C}_{\text{double}} + f_f \mathbf{C}_{\text{volume}}
$$

where $f_d$, $f_s$ and $f_f$ are the expansion coefficients. It is assumed that $\mathbf{C}_{\text{surf}}$, $\mathbf{C}_{\text{double}}$ and $\mathbf{C}_{\text{volume}}$ does not have (1,2), (2,1), (2,3) and (3,2) components due to the reflection symmetry property ($<S_{HV}S_{HV}^*> <S_{HV}S_{VH}^*> \approx 0$).

$$
\mathbf{C}_{\text{surf}} = \begin{bmatrix} |\beta|^2 & 0 & 0 \\ 0 & 0 & 0 \\ \beta^* & 0 & 1 \end{bmatrix},
\mathbf{C}_{\text{double}} = \begin{bmatrix} 1 & 0 & \alpha^* \\ 0 & 0 & 0 \\ \alpha & 0 & |\alpha|^2 \end{bmatrix},
\mathbf{C}_{\text{volume}} = \frac{1}{8} \begin{bmatrix} 3 & 0 & 1 \\ 0 & 2 & 0 \\ \frac{1}{3} & 0 & 3 \end{bmatrix},
$$

where $\alpha$ and $\beta$ are the polarization ratios between $HH$ and $VV$ of double and surface scatters. The decomposition results are obtained as

$$
P_s = f_s (1+|\beta|^2), \quad P_d = f_d (1+|\alpha|^2), \quad P_f = f_f.
$$

3.2 Modified three-component decomposition

If the azimuth rotation which means a rotation around the radar line of sight (LOS) is considered to a covariance matrix, the covariance matrix is changed as

$$
\begin{bmatrix}
\mathbf{C}(HV(\theta))\
\mathbf{C}(HV(\theta))^*\
\end{bmatrix} = [U_{\theta}]^T \begin{bmatrix}
\mathbf{C}(HV(\theta))\
\mathbf{C}(HV(\theta))^*\
\end{bmatrix} [U_{\theta}],
$$

$$
U_{\theta} = \begin{bmatrix} 
\cos^2 \theta & \sqrt{2} \sin \theta \cos \theta & \sin^2 \theta \\
-\sqrt{2} \sin \theta \cos \theta & \cos 2\theta & \sqrt{2} \sin \theta \cos \theta \\
\sin^2 \theta & -\sqrt{2} \sin \theta \cos \theta & \cos^2 \theta 
\end{bmatrix}
$$

The azimuth rotation affects the reflection symmetry property. For example, $<S_{HH}S_{HV}^*>$ and $<S_{HV}S_{VH}^*>$ in (19a) and (19b) are varied from zero as follows

a) Surface scattering case

$$
\langle S_{HH}S_{HV}^*(\theta) \rangle = \sin \theta \cos \theta \left[ \sin^2 \theta - |\beta|^2 \cos^2 \theta + \beta \cos^2 \theta - \beta^* \sin^2 \theta \right]
$$

$$
\langle S_{HH}S_{HV}^*(\theta) \rangle = \sin \theta \cos \theta \left[ \cos^2 \theta - |\beta|^2 \sin^2 \theta + \beta \sin^2 \theta - \beta^* \cos^2 \theta \right]
$$

b) Double-bounce scattering case

$$
\langle S_{HH}S_{HV}^*(\theta) \rangle = \sin \theta \cos \theta \left[ |\alpha|^2 \sin^2 \theta - \cos^2 \theta + \alpha^* \cos^2 \theta - \alpha \sin^2 \theta \right]
$$

$$
\langle S_{HH}S_{HV}^*(\theta) \rangle = \sin \theta \cos \theta \left[ |\alpha|^2 \cos^2 \theta - \sin^2 \theta + \alpha^* \sin^2 \theta - \alpha \cos^2 \theta \right]
$$

These influences appear in urban and mountain areas. For example, if a street pattern in urban area is not parallel to a direction of radar platform’s orbit, a ground-wall structure or wall is regarded to be rotated in the projection.
plane as shown in Fig. 3(a). Moreover, a slope area on mountain is considered that a flat area is tilted as shown in Fig. 3(b). Thus, three component scattering model proposed by Freeman and Du Run can not be applied to these areas where the reflectivity symmetry is not satisfied. Azimuth rotation angle $\theta$ in the projection plane that is perpendicular to range direction, can be estimated. The scattering matrix rotated by $\theta$ is expressed as

$$ \left[ \begin{array}{c} \mathbf{S} (\mathbf{H}(\theta)) \\ \mathbf{S} (\mathbf{V}(\theta)) \end{array} \right] = \left[ \begin{array}{c} \mathbf{S}_{\mathbf{H}} + j \mathbf{S}_{\mathbf{V}} \\ \mathbf{S}_{\mathbf{H}} - j \mathbf{S}_{\mathbf{V}} \end{array} \right]. $$

(24)

The elements of scattering matrix in circular polarization basis (LR) are derived following the transformation.

$$ S_{LL} = \frac{1}{2} \left( \mathbf{S}_{\mathbf{H}} - \mathbf{S}_{\mathbf{V}} + j 2 \mathbf{S}_{\mathbf{H}} \right), \quad S_{RR} = \frac{1}{2} \left( \mathbf{S}_{\mathbf{H}} - \mathbf{S}_{\mathbf{V}} + j 2 \mathbf{S}_{\mathbf{V}} \right) $$

If $S_{HV}$ in eq. (24) is assumed to be zero, $\text{Arg}(-S_{LL}S_{RR}^*)$ provides the rotation angle $\theta_{LL,RR}$.

$$ \text{Arg}(-S_{LL}S_{RR}^*) \bigg|_{\theta_{0} = \theta} \approx -4\theta $$

(26)

Thus, an approximated rotation angle $\theta$ in pixel of image can be estimated from the measured POLSAR data and inverse rotation (-$\theta$) can be done by eq. (21). The measured data which has an azimuth rotation angle is expressed as $<\mathbf{C}(\mathbf{H}(\theta))>$. The data after turning $<\mathbf{C}(\mathbf{H}(\theta))>$ to $-\theta$ is denoted as $<\mathbf{C}(\mathbf{H}(\theta))>$. Then a difference between $<\mathbf{C}(\mathbf{H}(\theta))>$ and $<\mathbf{C}(\mathbf{H}(\theta))>$ is calculated. In the case where the double-bounce scattering component is mainly rotated, the difference is derived as follows

Case A (double bounce is mainly affected by the rotation):

$$ \{ <\mathbf{C}_{\mathbf{HV}}(\theta)> - <\mathbf{C}_{\mathbf{HV}}(0)> \} \text{double:} $$

$$ \Delta C_{i1} = f_{i} \left[ (\cos \theta - 1) + 2 \Re (\alpha \sin \theta \cos \theta + |\alpha| \sin \theta \cos \theta) \right] $$

$$ - f_{i} \left[ |\alpha| \sin \theta \cos \theta + 2 \Re (\alpha \sin \theta \cos \theta + |\alpha| \sin \theta \cos \theta) \right] $$

$$ \Delta C_{i2} = f_{i} \left[ \sin \theta \cos \theta + 2 \Re (\alpha \sin \theta \cos \theta + |\alpha| \sin \theta \cos \theta) \right] $$

$$ - f_{i} \left[ \sin \theta \cos \theta + 2 \Re (\alpha \sin \theta \cos \theta + \alpha \sin \theta \cos \theta) \right] $$

$$ \Delta C_{i3} = 2 f_{i} \left[ 1 + |\alpha| \right] \sin \theta \cos \theta $$

$$ - 2 f_{i} \left[ 1 + |\alpha| \right] \sin \theta \cos \theta $$

$$ \Delta C_{i4} = f_{i} \left[ |\alpha| \sin \theta \cos \theta + |\alpha| \sin \theta \cos \theta + \alpha \cos \theta \right] $$

$$ - f_{i} \left[ |\alpha| \sin \theta \cos \theta + |\alpha| \sin \theta \cos \theta + \alpha \cos \theta \right] $$

$$ \Delta C_{i5} = \frac{\sqrt{2}}{f_{i}} \sin \theta \cos \theta $$

$$ + |\alpha| \cos \theta $$

$$ \Delta C_{i6} = \frac{\sqrt{2}}{f_{i}} \sin \theta \cos \theta $$

$$ + |\alpha| \sin \theta \cos \theta $$

$$ - f_{i} \left[ |\alpha| \sin \theta \cos \theta + |\alpha| \sin \theta \cos \theta \right] $$

$$ - f_{i} \left[ |\alpha| \sin \theta \cos \theta + |\alpha| \sin \theta \cos \theta \right] $$

(27a)

In eq. (27a), the volume component disappears, because (19c) is not affected by $\theta$. Similarly, I can derive the difference in the case where the surface scattering component is mainly rotated.

Case B (surface scattering is mainly affected by the rotation):

$$ \{ <\mathbf{C}_{\mathbf{HV}}(\theta)> - <\mathbf{C}_{\mathbf{HV}}(0)> \} \text{surface:} $$

$$ \Delta C_{i1} = f_{i} \left[ |\alpha| \sin \theta \cos \theta + \cos \theta \sin \theta \alpha \cos \theta \right] $$

$$ - f_{i} \left[ |\alpha| \sin \theta \cos \theta + \cos \theta \sin \theta \alpha \cos \theta \right] $$

$$ \Delta C_{i2} = f_{i} \left[ |\alpha| \sin \theta \cos \theta + \cos \theta \sin \theta \alpha \cos \theta \right] $$

$$ - f_{i} \left[ |\alpha| \sin \theta \cos \theta + \cos \theta \sin \theta \alpha \cos \theta \right] $$

$$ \Delta C_{i3} = 2 f_{i} \left[ 1 + |\alpha| \right] \sin \theta \cos \theta $$

$$ - 2 f_{i} \left[ 1 + |\alpha| \right] \sin \theta \cos \theta $$

$$ \Delta C_{i4} = f_{i} \left[ |\alpha| \sin \theta \cos \theta + |\alpha| \sin \theta \cos \theta + \alpha \cos \theta \right] $$

$$ - f_{i} \left[ |\alpha| \sin \theta \cos \theta + |\alpha| \sin \theta \cos \theta + \alpha \cos \theta \right] $$

$$ \Delta C_{i5} = \frac{\sqrt{2}}{f_{i}} \sin \theta \cos \theta $$

$$ + |\alpha| \cos \theta $$

$$ \Delta C_{i6} = \frac{\sqrt{2}}{f_{i}} \sin \theta \cos \theta $$

$$ + |\alpha| \sin \theta \cos \theta $$

$$ - f_{i} \left[ |\alpha| \sin \theta \cos \theta + |\alpha| \sin \theta \cos \theta \right] $$

$$ - f_{i} \left[ |\alpha| \sin \theta \cos \theta + |\alpha| \sin \theta \cos \theta \right] $$

(27b)
angle distribution of fig.6. Figure 8 shows the decomposition results estimated by proposed method. In fig 8, each contribution is calculated to be ratio as:

\[
\frac{P_i}{P_{Total}} = \frac{P_i}{P_s + P_d + P_v}
\]

Two urban areas are compared. The difference between two urban areas is an azimuth rotation angle estimated by eq (26). Urban area 1 is about 2.4 degrees. Urban area 2 is about -24.6 degrees. Four component scattering decomposition method and proposed method provide same result with respect to urban area 1. Since the azimuth rotation angle in this area is small, the differences of eq.(27) are almost zero. In urban area2, two methods provide the different results. Four component scattering decomposition method estimates that a volume scattering is dominant. The proposed method shows that a double bounce scattering is dominant.

5. CONCLUSIONS

This report presented a polarimetric calibration method using a particle swarm optimization. This calibration method can estimate all polarimetric calibration parameters including Faraday rotation angle and does not need a polarimetric data satisfying reflection symmetry property. Moreover, a polarimetric decomposition technique considering azimuth rotation based on PSO was proposed. I derived the differences of covariance matrix before and after compensating an azimuth rotation angle. These differences are used to solve an optimization problem. I applied the proposed decomposition method to ALOS/PALSAR data and compared the results of four component scattering decomposition method and proposed method. In urban area, the difference between two methods was confirmed.

6. REFERENCES


Fig. 4 Photograph of trihedral corner reflector.

Fig. 5 Response of trihedral corner reflector.

Table 1 Polarimetric calibration parameters estimated by proposed method with JAXA’s parameters.

<table>
<thead>
<tr>
<th></th>
<th>Proposed method (PSO, Nagasaki)</th>
<th>JAXA (Quegan method, Amazon)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Step 1</td>
<td>Step 2</td>
</tr>
<tr>
<td>$f_1$</td>
<td>0.70° -1.37°</td>
<td>0.70° -1.49°</td>
</tr>
<tr>
<td>$f_2$</td>
<td>0.99° -21.48°</td>
<td>0.99° -21.59°</td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>0.005° -163.7°</td>
<td>0.010° -131.48°</td>
</tr>
<tr>
<td>$\delta_2$</td>
<td>0.018° -158.4°</td>
<td>0.010° -128.11°</td>
</tr>
<tr>
<td>$\delta_3$</td>
<td>0.011° -157.2°</td>
<td>0.013° -79.37°</td>
</tr>
<tr>
<td>$\delta_4$</td>
<td>0.016° -164.0°</td>
<td>0.013° -151.50°</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>0.38°</td>
<td>0.03°</td>
</tr>
</tbody>
</table>

Fig. 6 HH image of ALOS/PALSAR

Fig. 7 Image of azimuth rotation angle.

(a) Surface scattering

(b) Double scattering

(c) Volume scattering

Fig. 8 Decomposition results by proposed method.