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Recalculation of air turborocket performance

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The turborocket engine has been proposed for use in very-high-speed vehicles, such as Earth-to-orbit launchers. The pure ramjet engine can't work at low Mach numbers, while the turbojet engine has poor performance at high Mach numbers because of the limitation of turbine inlet temperature. The turborocket engine improves weak points of both engines, it has one or two stage fans which work at low Mach numbers and turbines which use independent gas from free stream air. The turborocket engine has been introduced in the standard textbook of Dr. Jack L. Kerrebrock's "Aircraft Engines and Gas Turbines" , but unfortunately there are some misunderstandings and miscalculation of the engine performance. The engine performance of his book is too high. The text book is not revised though it used worldwide, so this paper is one of the proposal of correction of chapter 10.8 of the book.

1. The Air Turborocket

There are two type of the air turborocket , one version uses two liquid propellants, a fuel and an oxidizer, which are pumped to high pressure and burned in a gas generator , as indicated in the upper half of figure 10.26 (this figure are adopted from his book) . The fuel-rich combustion gas is expanded through a turbine, which drives the air compressor, and then combusted with the compressed air.

In another version, shown in the lower half of figure 10.26, liquid hydrogen is pumped to high pressure, vaporized and heated by heat exchanger, and then expanded through a turbine, which drives the air compressor, and then combusted with the compressed air.

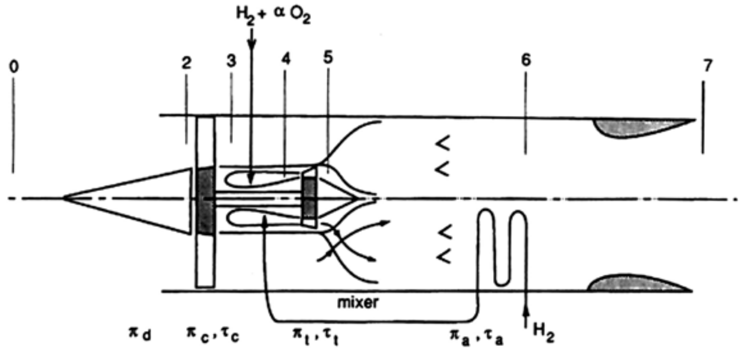

Figure 10.26
Air turborocket-bipropellant gas generator cycle (upper half of figure) and H_2 expander cycle (lower half of figure).

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The analysis of either version of the air turborocket proceeds similarly to that of the turbojet (refer to previous section of the book), but some qualitative differences arise from the separation of the compressor and turbine flows. Thus、the nozzle pressure and temperature ratios are given by

$$\frac{p_{t7}}{p_7} = \left(1 + \frac{\gamma-1}{2} M_7^2\right)^{\gamma/(\gamma-1)} = \delta_0 \pi_d \pi_c \pi_a \frac{p_0}{p_7} \qquad (1)(10.34)$$

Where p ; pressure, M ; Mach number, γ ; specific heat ratio, $\delta_0 = p_{t0}/p_0$, π_d=p_{t2}/p_{t0} diffuser pressure loss, $\pi_c = p_{t3}/p_{t2}$ compressor pressure ratio, π_a=p_{t6}/p_{t5} afterburner pressure loss. Suffix t means total condition, 7 means point number. The equation number (10.34) is original equation number of his book.

And

$$\frac{T_{t7}}{T_7} = \left(1 + \frac{\gamma-1}{2} M_7^2\right) = \theta_a \frac{T_0}{T_7} \qquad (2)(10.35)$$

Where θ_a=T_{t6}/T_0 is afterburner exit temperature ratio.

So that for an ideally expanded nozzle

$$M_7^2 = \frac{2}{\gamma-1}\left[(\delta_0 \pi_d \pi_c \pi_a)^{(\gamma-1)/\gamma} - 1\right] \qquad (3)(10.36)$$

And

$$\frac{T_7}{T_0} = \frac{\theta_0 \tau_c \tau_a}{1+[(\gamma-1)/2]M_7^2} = \frac{\theta_a}{(\delta_0 \pi_d \pi_c \pi_a)^{(\gamma-1)/\gamma}} = \frac{\theta_a}{\theta_0 \tau_c}$$

Where $\theta_0 = T_{t0}/T_0$ is inlet temperature ratio.The last equality applies only for the ideal cycle, without diffuser or combustor pressure losses.

The thrust per unit of airflow is given by

$$\frac{F}{\dot{m}_a a_0} = M_0\left(\frac{M_7}{M_0}\sqrt{\frac{T_7}{T_0}} - 1\right) = \sqrt{\frac{2}{\gamma-1}\frac{(\theta_0 \tau_c - 1)\theta_a}{\theta_0 \tau_c}} - M_0 \qquad (4)(10.37)$$

The specific impulse is

$$\frac{I}{a_0/g} = \frac{F/\dot{m}_a a_0}{\dot{m}_p/\dot{m}_a} \qquad (5)(10.38)$$

Where F ; thrust, I ; specific impulse, $\dot{m}_a$; air mass flow rate, $\dot{m}_p$; propellant mass flow rate, a_0 ; free stream sound velocity.

To compute the thrust and the specific impulse we must then determine the afterburner exit temperature ratio θ_a, the compressor temperature rise τ_c, and the ratio of propellant to air mass flows. The interrelationship of these is different for the bipropellant and H_2 expander versions, but they have some features in common.

In most cases, the performance of the turborocket will be limited by the pressure ratio available from the compressor, which is set by the available turbine power, so it will be desirable to operate the turbine at as high an inlet pressure as is practical. The limit will be set by stresses and temperatures in the turbine and in the combustion chamber. Thus, we regard p_{t4} and T_{t4} as design parameters of the turborockets, in the same sense that T_{t4} is a design parameter for the turbine engines.

The available compressor temperature rise is then determined by the turbine –compressor work balance:

$$\dot{m}_p \frac{p_{t4}-p_0}{\rho_p} + \dot{m}_a c_{pa}(T_{t3}-T_{t0}) = \dot{m}_p c_{pp}(T_{t4}-T_{t5}) \ , \tag{6}$$

Where c_{pa},c_{pp} being specific heat ratio of compressor and turbine gases. The first term on the left is the power required to pump the propellants, ρ_p being a mean density for the propellant mixture. Normally this term is small in relation to the compressor work, but it will be retained for now. Solving for the available compressor temperature rise, we get

$$\frac{p_0}{\rho_p}\left(\frac{p_{t4}}{p_0}-1\right) + \frac{\dot{m}_a}{\dot{m}_p} c_{pa} T_{t0}(\tau_c - 1) = c_{pp} T_0 \left(\frac{T_{t4}}{T_0} - \frac{T_{t5}}{T_0}\right)$$

$$\frac{\dot{m}_a}{\dot{m}_p} c_{pa} T_{t0}(\tau_c - 1) = c_{pp} T_0(\theta_t - \theta_5) - \frac{p_0}{\rho_p}\left(\frac{p_{t4}}{p_0}-1\right) \tag{7}$$

Introducing the perfect gas law; $p_0 = \rho_0 R_a T_o$, $R_a = c_{pa}(\gamma-1)/\gamma$,

$$\frac{\dot{m}_a}{\dot{m}_p} c_{pa} T_{t0}(\tau_c - 1) = c_{pp} T_0(\theta_t - \theta_5) - \frac{\rho_0}{\rho_p}\frac{\gamma-1}{\gamma} c_{pa} T_0 \left(\frac{p_{t4}}{p_0}-1\right) \tag{8}$$

$$\tau_c - 1 = \frac{\dot{m}_p}{\dot{m}_a}\left[\frac{c_{pp}}{c_{pa}}\frac{\theta_t}{\theta_0} - \frac{c_{pp}}{c_{pa}}\tau_c\theta_t\left(\frac{p_0}{p_{t4}}\right)^{(\gamma-1)/\gamma} - \frac{\rho_0}{\rho_p}\frac{\gamma-1}{\gamma}\frac{1}{\theta_0}\left(\frac{p_{t4}}{p_0}-1\right)\right]$$

$$\tau_c = \frac{1+\frac{\dot{m}_p}{\dot{m}_a}\left[\frac{c_{pp}\theta_t}{c_{pa}\theta_0} - \frac{\rho_0}{\rho_p}\frac{\gamma-1}{\gamma}\frac{1}{\theta_0}\left(\frac{p_{t4}}{p_0}-1\right)\right]}{1+\frac{\dot{m}_p c_{pp}}{\dot{m}_a c_{pa}}\theta_t\left(\frac{p_0}{p_{t4}}\right)^{(\gamma-1)/\gamma}} \tag{9}$$

which can be simplified considerably by noting that the second term in the denominator will normally be much less than unity, and that p_{t4}/p_0 is much greater than unity. Thus,

$$\tau_c = 1 + \frac{\dot{m}_p}{\dot{m}_a}\left(\frac{c_{pp}}{c_{pa}}\frac{\theta_t}{\theta_0} - \frac{\gamma-1}{\gamma}\frac{p_{t4}}{\rho_p R_a T_{t0}}\right) \tag{10)(10.39}$$

Above analysis is induced by Dr. J.K.Kerrebrock, but unfortunately we can't use the perfect gas law, because p_0 in equation (6) is the inlet pressure of propellant, not of air. Let p_p is the inlet pressure of propellant, equation (6) becomes

$$\dot{m}_p \frac{p_{t4}-p_p}{\rho_p} + \dot{m}_a c_{pa}(T_{t3}-T_{t0}) = \dot{m}_p c_{pp}(T_{t4}-T_{t5}) \tag{6'}$$

$$\frac{\dot{m}_a}{\dot{m}_p} c_{pa} T_{t0}(\tau_c - 1) = c_{pp} T_0(\theta_t - \theta_5) - \frac{p_p}{\rho_p}\left(\frac{p_{t4}}{p_p} - 1\right) \quad (7)'$$

$$\tau_c = \frac{1 + \frac{\dot{m}_p}{\dot{m}_a}\left[\frac{c_{pp}\theta_t}{c_{pa}\theta_0} - \frac{1}{c_{pa}T_{t0}}\frac{p_p}{\rho_p}\left(\frac{p_{t4}}{p_p} - 1\right)\right]}{1 + \frac{\dot{m}_p c_{pp}}{\dot{m}_a c_{pa}}\theta_t\left(\frac{p_0}{p_{t4}}\right)^{(\gamma-1)/\gamma}} \quad (9)'$$

Which can be simplified considerably by noting that the second term in the denominator will normally be much less than unity, and that p_{t4}/p_p is much greater than unity. Thus,

$$\tau_c = 1 + \frac{\dot{m}_p}{\dot{m}_a}\left[\frac{c_{pp}}{c_{pa}}\frac{\theta_t}{\theta_0} - \frac{1}{c_{pa}T_{t0}}\frac{p_p}{\rho_p}\frac{p_{t4}}{p_p}\right] = 1 + \frac{\dot{m}_p}{\dot{m}_a}\left[\frac{c_{pp}}{c_{pa}}\frac{\theta_t}{\theta_0} - \frac{1}{c_{pa}T_{t0}}\frac{p_{t4}}{\rho_p}\right] \quad (10)'$$

Equation (10) and (10)' seems to be the same, but it's occasional case caused by neglecting unity. Equation (9)' is correct and equation (9) is false.

2. H_2-Expander ATR

There is a release of chemical energy only in the afterburner in H_2-expander ATR, only an overall heat balance is required, in the form

$$\dot{m}_a c_{pa}(T_{t6} - T_{t0}) = \dot{m}_p h_{H2}$$

$$\frac{\dot{m}_p}{\dot{m}_a} = (\theta_a - \theta_0)\frac{c_{pa}T_0}{h_{H2}} \quad (11)(10.43)$$

Where h_{H2} is heat release of hydrogen. This relation may be regarded as determining $\dot{m}_p/\dot{m}_a$ if θ_a is prescribed. The afterburner temperature sets the hydrogen fuel flow, and then the temperature and pressure in the compressor drive turbine determine the compressor pressure ratio. Thus, the thrust per unit of air flow can be determine and also specific impulse. We determine some values of parameters as follows.

$c_{pa} = 1.004$ kJ/kgK , $c_{pp} = 14.2$ kJ/kgK , $\gamma = 1.4$, $T_0 = 226.5$K (assumed height of 30000m), $a_0 = 301.5$ m/s (assumed height of 30000m), h_{H2}=141947.2kJ/kg

$\theta_a = 10.0$, $\theta_t = 5.0$, $p_{t4} = 100$atm(10.13MPa)

The calculated values are described in the Table -1, and plotted in figure 1 and 2.

Table 1 Performance of Air Turborocket

M	1	2	3	4	5	6
θ_0	1.2	1.8	2.8	4.2	6	8.2
$\dot{m}_p/\dot{m}_a$	0.014	0.0131	0.0115	0.00929	0.0064	0.00288
τ_c	1.817	1.51	1.287	1.155	1.074	1.024
$F/\dot{m}_a a_0$	8.453	6.488	4.73	3.276	1.98	0.7959
$F'/\dot{m}_a a_0$	4.202	3.621	3.01	2.3	1.499	0.6366
I_{sp}	18575.6	15237	12653.9	10848.9	9518	8502.1
I'_{sp}	9233.9	8503.9	8052.4	7616.8	7205.8	6800.4

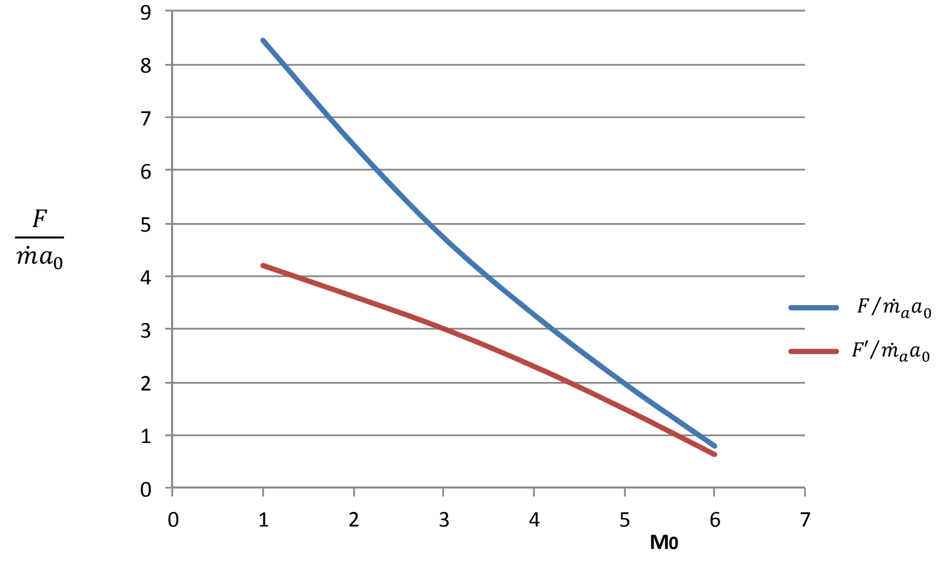

Figure 1 Thrust per unit of airflow for air-turborocket engines

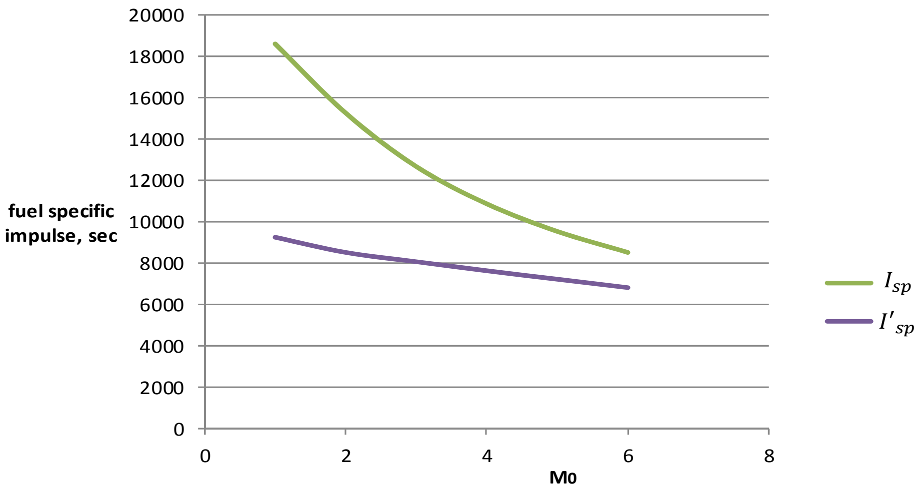

Figure 2 Fuel specific impulse for air turborocket engines with hydrogen expander cycle

Here $F/(\dot{m}_a a_0)$ is calculate by

$$\frac{F}{\dot{m}_a a_0} = \sqrt{\frac{2}{\gamma-1}\frac{(\theta_0\tau_c-1)\theta_a}{\theta_0/\tau_c}} - M_0 \qquad (12)$$

Dr. J.L.Kerrebrock might have made mistake, equation (12) is wrong, because denominator inside of the root should be $\theta_0\tau_c$, not θ_0/τ_c.
Correct thrust per unit of air flow $F'/(\dot{m}_a a_0)$ should be calculated by equation (4), namely

$$\frac{F'}{\dot{m}_a a_0} = \sqrt{\frac{2}{\gamma-1}\frac{(\theta_0\tau_c-1)\theta_a}{\theta_0\tau_c}} - M_0 \quad (13)$$

Using these F and F', specific impulse are calculated by

$$I_{sp} = \frac{F/\dot{m}_a a_0}{\dot{m}_p/\dot{m}_a}\frac{a_0}{g} \quad (14)$$

$$I'_{sp} = \frac{F'/\dot{m}_a a_0}{\dot{m}_p/\dot{m}_a}\frac{a_0}{g} \quad (15)$$

Obviously F and I_{sp} are too high comparing with F' and I'_{sp}. Figure 10.27 ($F/\dot{m}_a a_0$) and figure 10.28(fuel specific impulse, sec) of his book are presented for comparison .

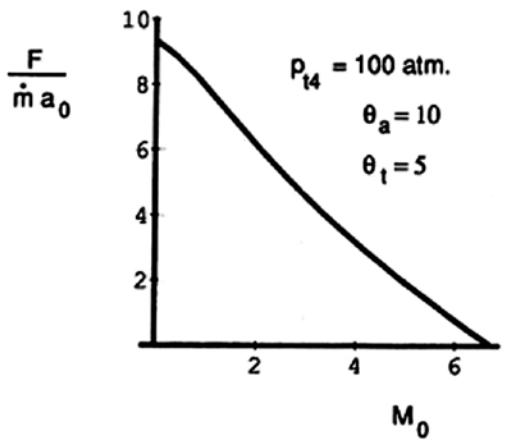

Figure 10.27
Thrust per unit of airflow for air-turborocket engines.

The values of thrust per unit of airflow, $F/\dot{m}_a a_0$, of figure 10.27 and figure 1 are almost same, so values of parameters used for the calculation, described in page 44, seems to be better selection. We also calculate the gas-generator cycle engine performance using same parameters. Thrust per unit of airflow of the gas-generator cycle engine is almost the same as H_2-expander cycle ATR, namely it is same miscalculation using the equation (12).

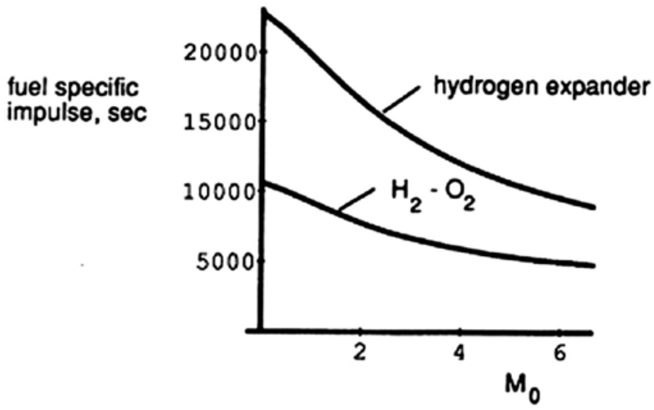

Figure 10.28
Fuel specific impulse for air-turborocket engines with hydrogen expander and H_2-O_2 gas generator cycles.

The fuel specific impulse of both cycle ATR of this figure are very high to use wrong $F/\dot{m}_a a_0$. The values of both engines should be almost half.

If we adopt $c_{pa} = 1.468\,\mathrm{kJ/kgK}$, h_{H2} =128165.0 kJ/kg in the equation (11), specific impulse will become smaller as described in reference book(1). It almost 4000sec at low speed, reasonable value with comparison the other air breathing engines like as a ramjet engine and a scramjet engine. The value of specific heat $c_{pa} = 1.468\,\mathrm{kJ/kgK}$ is at high temperature condition, heat release of hydrogen h_{H2} =128165.0kJ/kg is the net calorific values, not the gross calorific value. These values are better to use for the combustion calculation.

3. Conclusion

Miscalculation of the air turborocket performance of Dr.J.L.Kerrebrock is pointed out. His book "Aircraft Engines and Gas Turbines" is very high grade text book for all students and engineers. The chapter 10.8 should be revised to avoid the misunderstanding the performance of air turborocket.

4. References

(1) Koichi Suzuki "Jet Engines" written by Japanese, Morikita co. Tokyo, Japan, 2004