Rule Derivation for Arrival Aircraft Sequencing

Keywords: aircraft sequencing, optimization, sequencing rules, terminal area

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http://www.icas.org/ICAS_ARCHIVE/ICAS2010/PAPERS/180.PDF
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<thead>
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<th>Name</th>
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</thead>
<tbody>
<tr>
<td>Adriana Andreeva-Mori</td>
<td>Adriana Andreeva-Mori was born in Bulgaria in 1982. She received a bachelor’s degree, master’s degree and a Ph.D. in Aeronautics and Astronautics from The University of Tokyo in 2007, 2009 and 2012 respectively. She won the ICAS McCarthy Award for the best paper presented at the Student Session of the 28th International Congress of Aeronautical Sciences (ICAS) held in September 2012. Her research interests include air traffic management, trajectory optimization, path planning and aircraft sequencing.</td>
</tr>
<tr>
<td>Shinji Suzuki</td>
<td>Dr. Suzuki received his B.Eng., M.Eng. and Ph.D. degrees in Aeronautical Engineering from The University of Tokyo in 1977, 1979 and 1986 respectively. From 1979 to 1986 he worked as a researcher at Toyota Central R&amp;D Labs., Inc. He became an associate professor in Department of Aeronautics and Astronautics, the University of Tokyo in 1986. Since 1996 he is a full professor. He has been serving responsibilities in the University of Tokyo, as a director of Global Center for Innovation in Engineering Education and a director of Center for Aviation Innovation Research. He was a president of JSASS in 2011.</td>
</tr>
<tr>
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<td>Eri Itoh received a PhD from the Department of Aeronautics and Astronautics, The University of Tokyo in 2007. She worked at Eurocontrol Experimental Centre (EEC) in France as a doctoral researcher in 2006-2007. She started to work at Air traffic Management Department in Electronic Navigation Research Institute (ENRI) in 2007, and currently holds the position of senior researcher. She has been a guest researcher of National Aerospace Laboratory NLR in the Netherlands in 2008-2013, a visiting fellow with The University of Tokyo in 2012-2013, a visiting scholar of NASA Ames research center in 2013-2014, and a member of the program committee of the International Council of Aeronautical Science (ICAS) since 2010.</td>
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Rule Derivation for Arrival Aircraft Sequencing

Abstract
At present, the conventional way to sequence aircraft in the terminal area is to follow the first-come, first-served rule. Even though such sequencing is considered fair to all airlines and is associated with no increase in the workload of air traffic controllers, it is not always the optimal solution in terms of fuel burn and runway capacity. In this research we consider a substitute to the first-come, first-served rule leading to a more optimal sequencing which would reduce the total fuel burn when aggregated by all airplanes approaching a destination airport. The approach taken is to provide air traffic controllers with a simple guideline which can help them determine the sequence without increasing their workload too much and whenever possible add up to runway capacity. Sequencing is based on fuel burn simulations of single aircraft entering the terminal area of a sample airport. First, optimal aircraft sequences and their associated flight times under high-density operations are determined by Sequential Quadratic Programming. Next, the results are analyzed considering several attributes and three sequencing rules are proposed. Their effect is verified through Monte-Carlo simulations and it is concluded that through two simple swaps significant fuel savings of up to 17% of the extra fuel needed to make adjustments to the flight profile because of congestions can be achieved.
1. Introduction

1.1 Research Background and Objectives

Efficient scheduling of aircraft landings can improve runway throughput and reduce fuel burn. Currently, however, the most common conventional sequencing strategy is the first come, first served (FCFS) one, according to which aircraft are allowed to land in their order of arrival at the runway, i.e. the earlier the estimated time of arrival (ETA) is, the earlier the aircraft is going to get landing clearance. This rule has become so popular because of its simplicity and easy application which is a key factor for the workload of air traffic controllers. Another advantage of FCFS is that it is fair to all airlines since no preferences are executes. However, with the recent increase in air traffic, more importance has been placed on fuel burn and airport capacity and these factors need to be considered when determining the arrival sequence. Numerous systems aiding air traffic scheduling have been developed [1], [2], [3], but they all include hardware or/and software installation and staff training associated with the new tool. Furthermore, despite the notable advances in technology, air traffic control is likely to remain a human-centered operation for the foreseeable future. The terminal area where most re-sequencing occurs is a very dynamic environment, so air traffic controllers cannot afford to spend a long time determining an alternative sequence.

The research presented in this paper focuses on finding a substitute to the first-come, first-served rule which would reduce the total fuel burn during the descent. The approach taken is to provide air traffic controllers with a simple guideline which can help them determine the sequence without increasing their workload too much and whenever possible add up to runway capacity. Sequencing is based on fuel burn simulations of single aircraft entering the terminal area of a sample airport. First, optimal aircraft sequences and their associated flight times are determined by Sequential Quadratic Programming. Next, the results are analyzed considering several attributes and three sequencing rules are proposed. Their effect is verified through Monte-Carlo simulations and two simple swaps through which significant fuel savings can be achieved are presented.

The main contribution of this research is the revelation that operation improvements only can contribute to substantial fuel savings. What operation procedures are efficient depends on a number of factors such as traffic characteristics, aircraft types and airport terminal area. This research shows how such operation improvements can be easily determined, and the authors believe that this insight into sequencing procedures can be applied at virtually any airport worldwide.
1.2 Paper Organization

This paper is organized as follows: the simulation assumptions are presented in Section 2. They include description of the terminal area, traffic conditions, fuel burn modeling, operational constraints, such as minimum separation, precedence constraints and position shift constraints, and finally fuel burn evaluation, i.e. the parameter defined to evaluate each sequence presented later in the paper. Section 3 deals with optimal aircraft sequencing. First, under the assumptions described in Section 2, the fuel burn for a conventional sequencing is estimated and these results are shown in Section 3.1. These results are used as a reference for all other sequences proposed later in the paper. Next, optimal sequences are determined using Sequential Quadratic Programming and the results are presented in Section 3.2. Based on analysis of the optimal sequence, a search for rules is done (see Section 4). The extracted rules are verified in Section 5 and their possible effects are discussed in Section 6. This paper is summarized in Section 7.

2. Simulation Assumptions
2.1. Terminal Area

In this research the aircraft re-sequencing is performed in the terminal area.

![Waypoints in the terminal area considered](image)

**Fig. 1. Waypoints in the terminal area considered**

A sample airport (such as what used to be the approach configuration of Tokyo International Airport, the airport with the most passengers in Japan) is used for the basis of this research (Fig.1). After the aircraft enter the terminal area at one of the three
waypoints A, B or C, they are sequenced and exit at the final approach waypoint D. Aircraft have to be aligned with the runway, so they pass through the shaded area (shown with the triangle) before reaching point D. The incoming traffic should be merged before it is handed to the approach control, so all sequencing and spacing occurs in this area. Aircraft are usually directed following the brown dotted lines. Since the combined traffic from the south accounts for 70% of the total traffic, usually aircraft coming from the south are given priority and aircraft coming from the north are placed when there is an available slot in the waiting sequence.

2.2. Traffic Conditions
Traffic was simulated considering actual flow at Tokyo International Airport. Here, scenarios with 10 aircraft entering the terminal area in an interval of 13 minutes are generated. The ratio of heavy to medium aircraft is 1:1. Furthermore, 2 aircraft enter the terminal area at point A, 5 at waypoint B and 3 at waypoint C, which is proportional to the traffic volume at these three entry waypoints. These assumptions can adequately model the traffic at this airport in congested times.

2.3. Fuel Penalty for Delays
Every aircraft has an ideal descent time which minimizes the fuel burn. However, congestions in the terminal area often require changes in the descent time. The extra fuel burn incurred by positive or negative delays is often modeled as a combination of linear functions [4]. This research, however, uses a refined fuel burn model based on the optimization of single aircraft descent trajectories. The point mass aircraft model is used and constraints such as maximum allowed flight path angle (glide angle) of 3 deg are enforced. The descent trajectory is divided into stages during which the flight path angle, lift and thrust coefficients are constant. For the purposes of examining the most fuel-efficient sequencing in our numerical simulations, several new rules described below are introduced.

1) Whenever possible, the aircraft should fly the shortest distance between the waypoint at the entrance of the terminal area (A, B or C) and intermediate waypoint D1 (D2).
2) When the above is not feasible, flight time adjustment should be done by speed adjustment and/or vectoring (lengthening the flight path of the aircraft).
(3) The speed of the aircraft at waypoints A, B and C might vary, but the speed at the final approach waypoint D is fixed at 230 kt.
Simulations results confirmed that for each entry waypoint and aircraft type there is an ideal, optimal descent time which minimizes the fuel burn. However, since aircraft cannot always follow its optimal profile, constraints on the descent time are applied and the flight path for minimum fuel burn is determined. The graph showing the minimum fuel burn for various descent times can be seen in Fig.2.

![Fig. 2. Relations between the fuel burn and descent time](image)

The graphs in Fig.2 are created based on multiple optimizations with constrained descent time. Each of the markers in the figure corresponds to a simulation for a specific descent time. To illustrate the adjustments which need to be made in order to achieve the required descent time, several optimization examples are shown below. The aircraft considered is Boeing 737 medium aircraft entering the terminal area at waypoint A (Fig.1). The optimal descent time is found to be 700 s with a corresponding fuel burn of 1427 lb. The trajectory characteristics are shown in Fig.3. The aircraft maintains high altitude as long as possible so the flight path angle is zero for the first 90 s. After that the aircraft descents at its maximum allowed flight path angle of -3 degrees. The speed decreases gradually, too. The ground trajectory profile shows that aircraft follows the shortest distance allowed between the entry waypoint and the exit waypoint with lateral coordinates (0, 0).
Fig. 3. Descent parameters for flight time 700 s

For a flight time of 510 s, the aircraft follows the same flight path and the speeding up is achieved by increasing the speed in all stages. This requires extra thrust which results in increased fuel burn. Compared to the fuel burn of the optimal descent, the fuel burn for the 510-second-descent is 1748 lb, an increase of 22%.

Some delays can be compensated by speed adjustments, too. Consider a 750-second-descent shown in Fig. 4. The ground trajectory profile is the same as the one for the ideal descent shown in Fig. 3. The aircraft is delayed by decreasing the speed in all stages. The fuel burn is 1477 lb.

Fig. 4. Descent parameters for flight time 750 s
However, when the delay exceeds 810 s, the aircraft cannot follow the optimal descent path anymore or it would arrive at the final waypoint earlier than allowed. In such a case vectoring is necessary, as shown in the ground trajectory profile in Fig.5. The speed is lower than the optimal one, too. The fuel burn for the 900-second-descent is 21% higher than the fuel burn for the optimal descent.

Fig.5. Descent parameters for flight time 900 s

Simulations are performed for two types of aircraft, representatives of the heavy and medium category according to ICAO standards (to be discussed later in Section 2.4.1). It is also verified that around the optimal descent time, fuel burn can be modeled by a quadratic function with an error of less than 3.4 lb.

\[ f = a(t - t_{opt})^2 \]  

(1)

Here, \( f \) is the fuel burn increase, \( a \) is a parameter related to the aircraft type and entry waypoint altitude and distance from the final approach waypoint, \( t_{opt} \) is the absolute optimal flight time, i.e. the descent time that minimizes the fuel burn, and \( t \) is the actual flight time. The heavy aircraft have bigger values of \( a \) and the entry waypoints that are further from the final approach waypoint are associated with larger \( a \). Details on the optimization of single aircraft descents can be found in our previous work [5]. Therefore, instead of the commonly-used combination of linear functions, we model the fuel burn increment by a quadratic function.
2.4. Operational Constraints
2.4.1 Minimum Aircraft Separation

International Civil Aviation Organization (ICAO) has established minimum separation requirements to guarantee that aircraft do not suffer from the wake vortices induced by leading aircraft [6]. These separation requirements depend on the size of the aircraft pair, as shown in Table 1.

<table>
<thead>
<tr>
<th>Lead</th>
<th>Heavy</th>
<th>Follower</th>
<th>Medium</th>
<th>Light</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heavy</td>
<td>4 nm</td>
<td>5 nm</td>
<td>6 nm</td>
<td></td>
</tr>
<tr>
<td>Medium</td>
<td>3 nm</td>
<td>3 nm</td>
<td>5 nm</td>
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</tr>
<tr>
<td>Light</td>
<td>3 nm</td>
<td>3 nm</td>
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Table 1. ICAO separation standards

When performing the descent optimization of single aircraft trajectories, an assumption about the speed at the terminal area exit waypoint (the final approach waypoint) is done, i.e. all aircraft pass at waypoint D at speed of 240 kt. Therefore, the distance required minimum separation can be interpreted in seconds, instead of nautical miles.

<table>
<thead>
<tr>
<th>Lead</th>
<th>Heavy</th>
<th>Follower</th>
<th>Medium</th>
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<tr>
<td>Heavy</td>
<td>60 s</td>
<td>75 s</td>
<td>90 s</td>
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<tr>
<td>Medium</td>
<td>45 s</td>
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<tr>
<td>Light</td>
<td>45 s</td>
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Table 2. Minimum time separation at speed of 240 kt at the terminal area exit waypoint

Besides, at Tokyo International Airport, whose terminal area is considered in this research, no light aircraft are to be seen. Since we are looking for simple sequencing rules, the minimum time separation has been further simplified to just two values- 90 s and 60 s respectively, as shown in Fig.6. Even these separation standards are changed for some reason, as long as there is a separation difference among the aircraft classes, significant fuel gains are to going to be observed.
Fig. 6. Simplified required separation minimum

It is assumed that as long as the separation minimum between any two successive aircraft in a sequence is met, the minimum separation for all pairs of aircraft is also met.

Also, obviously at each of the entry points the separation requirements are met.

2.4.2. Precedence Constraints

Furthermore, certain precedence constraints are forced. Successive aircraft entering the terminal are at the same entry waypoint are not allowed to overtake each other, i.e. aircraft flying within the same jet route cannot swap positions in the final sequence. Similar assumptions were made by other researchers, so these are to be followed here, too [4], [7].

2.4.3. Position Shift Constraints

At present, the most commonly-used sequencing strategy is the first-come, first-served rule. However, it is not always the optimal one in terms of fuel burn and airport runway capacity. If a batch of aircraft consists of heavy and medium aircraft which are alternating in the sequence, the required minimum separation will be bigger than that for several heavy aircraft in a row, followed by several medium aircraft in a row, for example. Intuitively, this will result in delayed landing of the aircraft later in the sequence and thus overall reduced runway capacity. However, it is also unlikely to believe that in a batch of say 10 aircraft the last aircraft will come first in the adjusted sequence. Such a major change of the sequence will increase the workload of the air traffic controllers and most probably result in increased combined fuel burn of all aircraft. The terminal area of a busy airport is often congested so any suggested re-sequencing strategy should take into account the possible workload problems. In this research, the issue is tackled by introducing constrained position shifting [7]. We assume that an aircraft may be moved by no more than one position in the final sequence, i.e. the $i^{\text{th}}$ aircraft can land either on position $i-1$, $i$ or $i+1$.

Constrained position shifting has several advantages. First, since it does not change the sequencing too much, it is performed relatively easy. Second, it is still fair to all airlines
because no aircraft will be delayed by more than one position. Third, by putting constraints on the position shifts allowed, the number of possible sequences reduces greatly. This characteristic is of key importance for determining the optimal sequence.

In this research we consider batches of 10 aircraft, but to illustrate the possible sequences with constrained position shifting, an example for a batch of 5 aircraft is shown in Fig.7. Obviously, the same rules which govern the choice of possible positions in the final sequence for a batch of 5 aircraft apply to a batch of 10 aircraft, too.

Fig.7. For illustration purposes, the possible sequences for just five aircraft with constrained position shifting of 1 position are shown. The columns show the position in the final sequence, while the numbers in the boxes show the position of the aircraft in the FCFS sequence.

At the first position in the final sequence can be placed only the first or the second aircraft from the FCFS sequence. At position 2 in the final sequence there might come aircraft 1, 2 or 3 from the FCFS sequence. Consider the following sequence of the first three aircraft in the final sequence 1-2-4. The next aircraft can be either 3 or 5. For aircraft 3, the follower will be aircraft 5, so the final sequence will be 1-2-4-3-5. If the sequence is 1-2-4-5, though, no aircraft is left for the last position in the final sequence. Therefore, in this case, the branching 1-2-4-5 is impossible, so we are left with the only
option 1-2-4-3-5. Following the same logic the number of possible sequences for $n$ aircraft with constrained position shifting of 1 position can be determined by

$$Seq(n) = Seq(n-1) + Seq(n-2)$$

(2)

for $n > 2$

where

$$Seq(1) = 1$$

(3)

$$Seq(2) = 2$$

The possible sequences for $n$ aircraft can be divided into two blocks as shown in the upper and lower part in Fig. 7. The number of possible sequences in the upper block is the same as $Seq(n-1)$ and that in the lower block as $Seq(n-2)$.

On the other hand, if no constrained position shifting is considered and all permutations are taken into account, the number of possible sequences is $n!$. In our research we consider a batch of 10 aircraft, so with the constrained position shifting the number of possible sequences to be investigated is 89. If there were no position shift constraints, that number would be 3628800.

2.5. Fuel Burn Evaluation

In this research the objective function used to evaluate each sequence is related to the combined fuel burn by all ten aircraft. First, the first come, first served sequence is considered. If all aircraft could land at its estimated time of arrival, then the total fuel burn increase would be zero. We are interested only in the fuel burn increase inferred by any delays, being positive or negative, because only this fuel burn increase above the nominal one, i.e. the fuel burn penalty for delays, can be influence by any sequencing decisions. If FCFS sequence required some aircraft to be delayed, then this delays cause some fuel burn increase, which sum is defined as $fuel_{FCFS}$.

To evaluate any other sequencing, a new parameter $f_{par}$ is introduced. Suppose the total fuel burn increase for all ten aircraft for a certain sequencing is $fuel_{seq}$. In such a case, $f_{par}$ is defined as:

$$f_{par} = \frac{fuel_{seq} - fuel_{FCFS}}{fuel_{FCFS}}$$

(4)

In other words, $f_{par}$ shows how much fuel is necessary for the adjustments in a particular sequence compared to the fuel necessary when FCFS rule is applied. Positive values of $f_{par}$ indicate sequences which are worse than FCFS in terms of fuel burn and negative values indicate sequences which result in fuel saving compared to FCFS.
3. Optimal Sequencing

3.1. First Come, First Served Sequence

Here, only the static case is considered, i.e. in each simulation we have full knowledge of all 10 aircraft, i.e. their expected arrival time (ETA) and their type (medium or heavy) are known. For systems aiming at real-time optimization such an assumption is a constraint, but since we are going to use the optimization results just to extract rules, the static case is completely sufficient.

First, the FCFS arrival sequence is considered and the necessary flight time adjustments are made to meet the separation requirements discussed in Section 2.4.1. At this point, aircraft are not required to land earlier than their estimated time of arrival even if such a change would not infringe the separation minimum with the leading aircraft. This assumption reflects the common FCFS execution at most airports. Once the necessary time adjustments are determined, the fuel burn increase $\text{fuel}_{\text{FCFS}}$ is calculated based on the results obtained by single aircraft descent optimization shown in Section 2.3. $\text{fuel}_{\text{FCFS}}$ varies in each scenario, but on average it is about 12% of the total fuel burnt during the descent in the terminal area.

Fig.8. Aircraft sequencing according to FCFS, i.e. aircraft are scheduled according to their ETA by simply applying the required minimum separation between them

3.2. Optimal Sequence

Next, the optimal sequence for each scenario and the associated flight time adjustments are found. This is done as follows. The 89 possible sequences generated for a batch of 10 aircraft with a maximum allowed position shift 1 are considered (see Section 2.4.3). For each sequence, optimization of the flight times of all ten aircraft is performed using Sequential Quadratic Programming (SQP) [8], [9], [10]. Because of the nature of the optimization, if no precedence constraints are imposed, i.e. if the possible sequences are not generated beforehand and a general solution is sought, in most cases the program gets trapped into a local minimum and there is no guarantee that the obtained sequence is the best one. To deal with this problem we look into all 89 possible sequences and vary just the flight times looking for the combination which will
minimize the total fuel burn for all aircraft. Once the minimum fuel burn for each sequence is determined, these 89 values are compared and the minimum one is chosen as the best sequencing candidate. The fuel burn increase associated with this sequencing is written as fuel\text{opt}. Finally, fuel\text{opt} is compared to fuel\text{FCFS}. A histogram of the results for Monte-Carlo simulations for 100 scenarios is shown in Fig.9.

![Histogram of fuel savings by optimal sequencing of 10 aircraft.](image)

**Fig.9. Fuel savings by optimal sequencing of 10 aircraft.**

The horizontal axis shows the fuel parameter \( f_{\text{par}} = \frac{\text{fuel}\text{opt} - \text{fuel}\text{FCFS}}{\text{fuel}\text{FCFS}} \) in percentage, i.e. how much fuel is necessary for the optimal adjustments compared to the fuel necessary when FCFS rule is applied.

As seen from Fig.6, even though in 32% of the cases almost no fuel savings were observed, in the remaining 68% improvements of up to 80% of the extra fuel needed to compensate for the congestion when FCFS is applied.

Next, the number of swaps in each scenario is investigated. For example, when the optimal sequence is 1-2-4-3-5-6-7-8-9-10, there is only one swap between the positions of aircraft 3 and aircraft 4, when the optimal sequence is 2-1-4-3-5-6-7-8-9-10 there are two swaps, one between 2 and 1 and another one between 3 and 4. For 10 aircraft the maximum number of swaps is 5 and happens if the optimal sequence is \([2-1]-[4-3]-[6-5]-[8-7]-[10-9]\), where the swapped pairs are shown in brackets. The number of swaps per scenario is shown in Table 3.

<table>
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<tr>
<th>100 scenarios</th>
<th>0 swaps</th>
<th>1 swap</th>
<th>2 swaps</th>
<th>3 swaps</th>
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<td>Scenarios</td>
<td>36</td>
<td>40</td>
<td>18</td>
<td>6</td>
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**Table 3. Number of swaps in the optimal sequencing for 100 scenarios**
In one third of the cases the optimal sequence if the one decided by the FCFS rule, but in 40% of all cases one swap minimizes the total fuel burn.

4. Sequencing Rules Extraction

If we want to suggest some intuitive re-sequencing rules, though, knowing the number of swaps only will not be enough. It is important to investigate the kind of FCFS configurations subject to swaps so that air traffic controllers can be presented with a guide helping them to decide when and which aircraft to swap. Here, the swaps are divided in 8 types based on the size of aircraft included in the swap and the two aircraft preceding the pair and following the pair. The number of aircraft in each configuration is chosen to be 4 because when a pair of aircraft is swapped, it affects the separation time required to the preceding and the following aircraft. For example, if the 4th and the 5th aircraft in the sequence are swapped, we look at the size of aircraft 3, 4, 5 and 6. The 8 types of swaps are shown in Table 4. By investigating these 8 swap configurations, all cases of aircraft size can be considered as under our assumptions each configuration can be treated as an isolated one and swaps of the middle two aircraft do not affect the required minimum separation between the aircraft preceding the first one and the one following the fourth one.

We are not interested in swaps of aircraft of the same size since it is expected that such swaps will lead to just minor improvements in the total fuel burn because the coefficients characterizing the fuel burn increase $a$ (discussed in Section 2.3) for same-sized aircraft are very similar.

Several observations on the required separation can be made. Consider the minimum time required to land all four aircraft $t_{four}$. $t_{four}$ decreases by 30 sec for swaps type 2 and type 8, increases by 30 sec for swaps type 4 and type 6 and does not change for other swaps. In other words, swaps 2 and 8 improve both fuel burn and runway capacity, while swaps 4 and 6 might improve the fuel burn, but would result in decreased runway capacity.

To investigate the type of swaps in more detail a new series of Monte-Carlo simulation for 1000 random scenarios is conducted. The analysis approach taken is slightly changed. All configurations shown in Table 4 are investigated. A sample sequencing configuration is shown in Figure 10.
Table 4. Swap types depending on the size of the swapped pair

<table>
<thead>
<tr>
<th>Configuration</th>
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<td><strong>FCFS</strong> 60</td>
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<td>90</td>
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<tr>
<td><strong>Swap</strong> 60</td>
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<tr>
<td><strong>Swap</strong> 60</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig.10. A sample configuration used to analyze the type of swaps performed to obtain the best sequence which minimizes the total fuel burn

In this case, the optimal configuration is 1-3-2-4-5-6-7-8-9-10, a one swap scenario. It should be noted that most optimal sequencing scenarios included just a single swap. As you go through the FCFS sequence, you first isolate the swapped pair and two aircraft around it, in this case 1-3-2-4. This is a swap type 3 according to Table 4. The next group of 4 aircraft is of type 1, but there is no swap here. Next comes a group of type 5, followed by a group of type heavy-medium-medium-heavy. Since for the groups
at the beginning and the end of the sequence there are no four aircraft to form the group, all possibilities are considered, so we count a group of type 1 and 6. As a result, the analysis of this sequencing is one “swap” type 1, two “no swap” type 1, one “no swap” type 5, one “no swap” type 6. A similar analysis was done for 1000 scenarios randomly generated. The results are summarized in Table 5.

<table>
<thead>
<tr>
<th>Type</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Swap</td>
<td>132</td>
<td>294</td>
<td>61</td>
<td>30</td>
<td>60</td>
<td>6</td>
<td>0</td>
<td>155</td>
</tr>
<tr>
<td>No swap</td>
<td>460</td>
<td>180</td>
<td>113</td>
<td>479</td>
<td>492</td>
<td>541</td>
<td>196</td>
<td>308</td>
</tr>
</tbody>
</table>

Table 5. Number of swapped and non-swapped 4-aircraft groups for 1000 scenarios

Swaps of type 2 and 8 are of the greatest interest not only because they are dominant among the swapped pairs, but also because such swaps would result in a longer sequence of aircraft of the same size uninterrupted by aircraft of other size.

The next step is to determine under what conditions aircraft in configuration type 2 and type 8 are swapped. To do so, several attributes of the configuration are investigated. They are shown in Figure 11. ETA is the estimated time of arrival, i.e. the flight time which would minimize the fuel burn had there been no other interfering aircraft. Available time of arrival is the time which would be required in the FCFS sequence considering the earliest time at which the first aircraft in the configuration can land, i.e. the earliest available arrival time. This accounts for possible delays carried over from the previous configurations.

Fig.11. Attributes of the aircraft configuration which might influence the swapping

The author is aware that interaction between the attributes is very likely, but the conditions for swapping need to be simple and straightforward so an approach such as neural network is not appropriate. The proposed attributes are considered in different combination pairs and the optimization results are analyzed. However, satisfying results
are obtained only for swaps type 2 with attributes $a_{i1}$ and $a_{i2}$. Swaps type 8 cannot be analyzed well using a simple combination of the above attributes. The results of type 2 swap analysis are shown in Figure 12. The green dots represent swapped aircraft pairs and the blue crosses represent the non-swapped aircraft pairs. It can be seen that more swaps occurred when the first three aircraft in the sequence were relatively close to each other.

![Swapped and non-swapped pairs](image)

**Fig.12. Swapped and non-swapped pairs**

5. Sequencing rules

Based on the results discussed in the previous section, the following three rules were formulated and their effect on fuel burn was investigated through Monte-Carlo simulations. Here, ETA is simply the optimal descent time for each aircraft had no other aircraft been in the airspace at that time. In the conventional scheduling, the sequencing is kept unchanged and only the arrival times are adjusted.

**Rule 1**
Swap the $i^{th}$ and the $i+1^{th}$ aircraft if:
1.1) they are part of configuration type 2
1.2) $(ETA(i+1)-ETA(i))+(ETA(i)-ETA(i-1))<120 \ [s]$

**Rule 2**
Swap the $i^{th}$ and the $i+1^{th}$ aircraft if:
2.1) they are part of configuration type 2
2.2) $(ETA(i+1)-ETA(i))+(ETA(i)-ETA(i-1))<120 \ [s]$ AND $ETA(i)-ETA(i-1)<60 \ [s]$

**Rule 3**
Always swap the $i$th and the $(i+1)$th aircraft if they are part of configuration type 8. The essence of these rules is shown in Table 6.

<table>
<thead>
<tr>
<th>Rule</th>
<th>Configuration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rule 1</td>
<td>ETA less than 120 s</td>
</tr>
<tr>
<td></td>
<td>Swap 60 90 60</td>
</tr>
<tr>
<td>Rule 2</td>
<td>ETA less than 120 s</td>
</tr>
<tr>
<td></td>
<td>less than 60 s</td>
</tr>
<tr>
<td></td>
<td>Swap 60 90 60</td>
</tr>
<tr>
<td>Rule 3</td>
<td>ETA</td>
</tr>
<tr>
<td></td>
<td>Always swap 60 60 60</td>
</tr>
</tbody>
</table>

Table 6. Rules extracted from analyzing batches of 4 aircraft. ETA stands for estimated time of arrival, the figures “60” and “90” show the required separation.

5.1. Rule 1

In the Monte-Carlo simulations the aircraft are required to land as early as possible in order to maximize the runway capacity. The results for 1000 cycles are shown in Figure 13. 138 swaps are performed with average fuel parameter $f_{par}$ of -11.1%.

![Fig.13. Fuel improvements by the introduction of Rule 1](image)
5.2. Rule 2

In a manner similar to Section 5.1, the effects of Rule 2 are verified in Monte-Carlo simulations and the results are shown in Fig. 14. Here, compared to the 138 swaps performed with Rule 1, there are only 96 swaps. The average fuel parameter $f_{par}$ is -11.8%, or just slightly better than that of Rule 1. Rule 2 results in fewer swaps with more fuel savings, but the rule itself is more complicated than Rule 1, so taking this into account we conclude that the simpler Rule 1 excels overall.

![Fig.14. Fuel improvements by the introduction of Rule 2](image)

5.3. Rule 3

The effects of Rule 3 were analyzed not only in terms of fuel burn improvements, but also in regard of the runway capacity by considering the arrival time of the last aircraft in the group. The results from Monte-Carlo simulations are shown in Fig. 15 and Fig. 16. This rule could not be extracted very accurately from the optimal results, i.e. swaps are made even at places where they should not be made. Even so, the fuel gains from the appropriately swapped aircraft exceed the fuel losses by the inappropriate swaps and the average $f_{par}$ is -12.3%, higher than expected. Besides, the arrival time of the last aircraft in the group was on average 35 s earlier than that in the case of FCFS, which means that Rule 3 not only decreases the total fuel burn, but increases runway capacity, too.
When Rule 1 and Rule 3 are combined and applied simultaneously, on average, the last aircraft lands 34.6 s earlier than in the FCFS sequence and the fuel parameter is -17%. The histograms of these results are shown in Fig.17 and Fig.18.
Fuel savings by the simultaneous application of Rule 1 and Rule 3 are measured by a fuel parameter of -17%, which is twice less than that by the optimal sequencing discussed in earlier and is less than the sum of fuel savings of Rule 1 and Rule 3 applied independently. Several possible reasons might be behind these numbers. First, in the optimal solution the flight time can be adjusted very precisely to minimize the fuel burn and no aircraft arrives uselessly early. The flight time adjustments might play just an important role in the fuel burn as the sequencing itself. Next, there are aircraft configurations which might be subject to both Rule 1 and Rule 3 re-sequencing, but because the possible re-sequencing groups overlap, only one of the rules is applied. Therefore, even though the results obtained are not optimal, they are better than the widely-used first-come, first-served rule.
6. Discussion

6.1. Equity between aircraft types

Equity between aircraft types is not explicitly stated in the rules formulated above. The rules might not be feasible if they prioritize only medium or only heavy aircraft. It is difficult to evaluate and compare the importance of heavy and medium aircraft so here an investigation of the fuel parameter for medium and heavy aircraft separately is conducted. When applying Rule 1 and Rule 3 simultaneously, the fuel burn difference relative to the extra fuel burn for heavy aircraft is shown in Fig.19 and for medium aircraft- in Fig.20. The average fuel parameters in both cases are -12% and -22% respectively. Therefore, both heavy and medium aircraft benefit from the new sequencing rules.

![Fig.19. Fuel improvements for heavy aircraft](image1.png)

![Fig.20. Fuel improvements for medium aircraft](image2.png)
6.2. Financial and Environmental Impact

Next, the financial and environmental impact of the new rules is calculated. The environmental impact calculations are based on the carbon emissions calculator developed by ICAO [11]. By burning a kilogram of aviation fuel, 3.16 kg of CO\textsubscript{2} is emitted. The 17% savings discussed in Section 1.1.1 are equal to 576 kg aviation fuel for 10 aircraft entering the terminal area. Assume that there are eight such groups of ten aircraft, or approximately 2 hours of relatively congested airspace every day. The fuel savings would then be equal to 4,608 kg daily, or 1,681,920 kg annually. Assume that the price of jet fuel is USD3.13 per gallon [12], which is USD0.83 per liter. The density of aviation fuel is 0.81 kg/l, so one kilogram of aviation fuel costs about USD1. Therefore, the annual savings from the fuel reduction only would be USD1,681,920.

The environmental impact can be expressed as number of trees needed to absorb the carbon dioxide which would be emitted if the conventional sequencing were followed. According to data released by the Japanese Ministry of Agriculture, Forestry and Fisheries [13], one Japanese cedar absorbs about 14 kg CO\textsubscript{2} per year [14]. Therefore, the number of trees needed to absorb 1,681,920 kg is 120,137 trees. Assuming that 460 trees are planted on 0.5 ha and the size of a standard soccer field is 70 m per 110 m [15], the equivalent of more than 166 soccer fields planted with Japanese cedars are needed to set off the CO\textsubscript{2} released by aircraft. In other words, implementing the proposed sequencing rules is equivalent to planting 166 soccer fields covered with Japanese cedar every year.

6.3. Applications at Other Airports

In this research the terminal area was modeled based on Tokyo International Airport, but investigating other airports can be done in a similar manner. When conducting such a study, the following steps have to be followed:

1. Define terminal area, including entry points, exit points, as well as spatial and possible speed constraints at each waypoint. This should be done in close cooperation with air traffic controllers and pilots to ensure a realistic terminal area model.
2. Model the traffic, considering the type of aircraft landing at the airport and the number of aircraft in congested and non-congested times for each entry point.
3. Model the fuel burn of each aircraft type and each entry point by optimizing descent trajectories with descent time constraints. This step is equivalent to creating a database for fuel burn for aircraft at this particular airport.
4. Find the optimal sequences for a sufficient number of scenarios. If flight data is available, this might increase the reliability of the sequencing rules even more.
5. Search for “useful swaps” which reduce total fuel burn in the optimal sequences and extract sequencing rules.
6. Verify the statistical performance of the above rules through Monte-Carlo simulations.
7. Check for airline/aircraft type equity.
   Proposing these concrete steps which can be applied to practically any hub airport is considered a major contribution of this research.

7. Conclusion and Future Work
   This research proposes guidelines for aircraft sequencing in order to reduce the fuel burnt by aircraft during their descent in the terminal area. First, optimal sequencing based on descent trajectories minimizing aircraft fuel burn are computed. These sequences are analyzed and knowledge about the kind of swaps made is extracted.
   Three rules are proposed and their efficiency is verified by Monte-Carlo simulations of groups of 10 aircraft. It is concluded that if the rules summarized in Fig.21 are applied, the fuel burn increase caused by terminal area congestions under high-density operations can be decreased by 17% and the time necessary to land all 10 aircraft can be shortened by 34 s. The exact figures depend on the capacity, number of approaching aircraft and separation policies at each airport, but as long as there is a difference in the required minimum separation between different aircraft classes, resequencing is expected to result in fuel burn abatement.

![Fig.21. Rules defining the most efficient swaps](image)

An investigation which was not conducted in this research is the verification of the
effects of the proposed sequencing rules when they are presented to air traffic controllers. The authors argue that seemingly simple swaps can prove to be highly efficient for fuel savings, but at present there is no practical way to evaluate their actual simplicity in terms of workload increase, for example. Besides, since a considerable uncertainty is associated with the rules extraction, it is important to see if the potential benefits will be meaningful enough under various scenarios. Therefore, to investigate the proposed sequencing rules, their implementation in an air traffic control simulator, followed by experiments including air traffic controllers are necessary.

Furthermore, even though the rules proposed perform well on average, their Monte-Carlo simulation statistical verification showed that there would be aircraft which would “suffer” from the new rules and increase their fuel burn. To avoid extreme case, “exception rules” are necessary, i.e. further investigation of the scenarios which resulted in extreme fuel burn losses is recommended.

Another necessary verification is investigation of the rules for different terminal areas and traffic conditions, such as the variation of heavy to medium aircraft, number of aircraft per hour, and airline preferences. A quick simulation not presented in this dissertation has shown that the rules are sensitive to traffic conditions, so it would be interesting to find the thresholds when new rules are necessary. It is interesting to see whether the suggested rules will change under the “best-equipped, first-served” policy considered by FAA [16], too.

The authors believe that the guidelines suggested are simple enough and therefore easily applied. It is the authors’ sincere hope that this work would provide some useful insights for environmentally-friendly and economically-beneficial aircraft sequencing.

Acknowledgements
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[16] Federal Aviation Administration, "NextGen Implementation Plan," 2010