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Numerical Study of Transonic Flutter of a Two-Dimensional Airfoil

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A Two-dimensional Airfoil

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ABSTRACT

Transonic flutter characteristics of a NACA64A010 airfoil with two-degrees-of-freedom are investigated theoretically. For this purpose, an unsteady aerodynamic code based on the transonic small perturbation equation, which can be applied for the wide range of the reduced frequency based on semi-chord (0 \leq k \leq 0.5) and Mach number (from subcritical to above Mach 1), has been developed. The finite difference scheme employed in the code is a time-marching, semi-implicit and implicit two-sweep procedure. Flutter calculations are performed for two typical binary systems, one of which simulates the vibrational characteristics of a typical streamwise section of a sweptback wing, and the other of which simulates that of an unswept wing. A sharp transonic dip of the flutter boundary has been predicted for the former case while the relatively mild dip for the latter. For the purpose of identifying the possible mechanism of the transonic dip phenomenon, examinations are made of not only the flutter modes and frequencies but also the shock wave patterns and the unsteady load distributions at each Mach number corresponding to the flutter boundary. As a result of these examinations, it is concluded that the mechanism of the single-degree-of-freedom flutter, which is caused by the large negative damping produced by the phase lag of the shock wave motion, is dominating the flutters at the bottom of the transonic dip when the mass ratio is relatively large.

NOMENCLATURE

\( a \) = distance of elastic axis behind mid-chord, percent semichord
\( b \) = semichord
\( C \) = chord length
\( C_L \) = lift coefficient \((L/(\frac{1}{2}\rho_\infty U_\infty^2C))\)
\( C_{L,1} \) = first harmonic (in complex form) of lift coefficient
\( C_{L,1,\theta} \) = first harmonic (in complex form) of lift coefficient per unit \( \theta \)
\( C_M \) = pitching moment coefficient \((M_p/(\frac{1}{2}\rho_\infty U_\infty^2C))\)
\( C_{M,1} \) = first harmonic (in complex form) of pitching moment coefficient (positive nose up)
\( C_{M,1,\theta} \) = first harmonic (in complex form) of pitching moment coefficient (positive nose up) per unit \( \theta \)
\( C_p \) = pressure coefficient \((p-p_\infty)/(\frac{1}{2}\rho_\infty U_\infty^2))\)
\( C_p^* \) = pressure coefficient at sonic condition
\( \Delta C_p \) = pressure loading coefficient \((C_p^{(L)}-C_p^{(U)})\)
\( C_{p,\theta} \) = first harmonic component (in complex form) of local pressure coefficient per unit \( \theta \)
\( \Delta C_{p,\theta} \) = first harmonic component (in complex form) of local pressure loading coefficient per unit \( \theta \)
\( f(x) \) = \( y \)-coordinate of the time mean contour of the airfoil
\( f^{(\theta)}(x,t) \) = unsteady displacement of airfoil surface from its time mean contour
\( g \) = structural damping coefficient unless otherwise noted
\( h \) = plunging displacement of elastic axis (positive down)
\( I_s \) = moment of inertia per unit span about elastic axis
\( IZ \) = total number of grid points in \( z \) direction
\( JZ \) = total number of grid points in \( y \) direction
\( k \) = reduced frequency \((boa)/U_\infty\)
\( L \) = total lift per unit span (positive up)
\( L_1, L_2 \) = aerodynamic coefficients for flutter calculation (see Eq. 3-11)
\( M_\infty \) = freestream Mach number
\( M_l \) = local Mach number
\( M_s \) = pitching moment per unit span about

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1. INTRODUCTION

The transonic flutter has been a major aeroelastic problem for designing modern aircrafts since many experimental evidences have shown that the transonic regime is most critical for flutter-free requirement of wing surfaces\(^9\). Because of the lack of reliable methods to predict transonic unsteady aerodynamic loadings, the determination of the flutter boundary has been relied on experiments using costly aeroelastic models.

Recently, the several finite-difference methods for predicting unsteady transonic flow field, including shock wave, around an oscillating airfoil and wings have been presented\(^1\)\(^-\)\(^3\). Among these methods, those based on the transonic small perturbation equation are most efficient, and hence are beginning to be applied to the flutter calculations of two-dimensional airfoils and a rectangular wing. Yang, et al\(^1\)\(^4\) applied the relaxation program STRANS and UTRANS of Ref. 12 and the indicial method (LTRAN2) of Ref. 15 to the flutter analyses of NACA64A006 and NACA64A010 airfoils with two-degrees-of-freedom. They examined the effects of various parameters such as mass ratio, center of mass location, frequency ratio etc. on the flutter speed. Mach numbers considered were 0.7–0.85 for NACA64A006 airfoil and 0.72–0.80 for NACA64A010 airfoil. A flutter analysis of a rectangular wing having circular-arc airfoil section was performed by Eastep and Olsen\(^1\)\(^6\) using the three-dimensional, relaxation program TDUTRN of Ref. 12 at Mach numbers 0.85 and 0.875. In these flutter analyses, the conventional U-g method was employed.

Another approach to the transonic flutter analysis is to calculate the aeroelastic responses of an airfoil by simultaneously integrating the structural equations and the governing aerodynamic equations. Such an approach was first taken by Ballhaus and Goorjian\(^1\)\(^7\). They performed the aeroelastic response analysis of a NACA64A006 airfoil with a single pitching-degree-of-freedom using their own code LTRAN2 for unsteady transonic flow calculations. Rizzetta\(^1\)\(^7\) extended the procedure of Ref. 15 and calculated the aeroelastic responses of NACA64A010 airfoil in three-degrees-of-freedom (plunging, pitching and aileron pitching) at Mach numbers 0.72 and 0.82. The aeroelastic response behavior of NACA64A006 in a single- and two-degrees-of-freedom at Mach numbers 0.85 and 0.88, was also studied by Yang, et al\(^1\)\(^8\) by using the similar procedure. Both Rizzetta and Yang, et al employed LTRAN2 code for the unsteady transonic flow calculations.
As mentioned above, the transonic flutter calculations reported so far employed the relaxation program UTRANS (or TDUTRN)\textsuperscript{11,12} or the time integration program LTRAN2, or both for unsteady aerodynamic force calculations. Yang, et al. pointed out in Ref. 14 that these aerodynamic codes (UTRANS, TDUTRN and LTRAN2) have considerable limitations on their applicable range of the reduced frequency. For example, in Ref. 14, the reduced frequencies $k(=b_0/U_\infty)$ considered were never higher than 0.15 when UTRANS was used, due to the “frequency limitation problem”\textsuperscript{19,20} which is inherent to the relaxation method, and were not higher than 0.1 when LTRAN2 was used because of the low-frequency approximation used in LTRAN2. The Mach numbers considered in these reports were also limited to the low transonic Mach numbers.

In order to understand the flutter characteristics of the airfoils in the entire transonic Mach number range and in the wide range of various parameters, the limitations on the reduced frequency and Mach number in these transonic codes must be removed. Recently, Houwink and van der Vooren\textsuperscript{20} have presented the modified version of LTRAN2 code (LTRAN2-NLR), in which they added the time derivative terms ($\phi_t$ and $f_t^{(i)}$) neglected in the original code both in the flow tangency condition and in the expression for the pressure coefficient (and hence the wake boundary condition)\textsuperscript{1}. It is reported that the direct applicability of LTRAN2 is improved typically from $k\leq0.15$ to $k\leq0.40$. Custon and Angelini\textsuperscript{21} have also developed their own code, based on almost the same mathematical model as LTRAN2-NLR. Flutter analysis using these improved codes has not yet been reported.

In this report, a new transonic small perturbation code, which can be used for the wide range of the reduced frequency ($k=0.0001$) and for the entire transonic Mach number range (from subcritical Mach numbers to over Mach number 1), is developed in section 2. Then, the unsteady aerodynamic forces calculated by this code are applied for the flutter analysis of NACA64A010 airfoil with two-degrees-of-freedom in section 3. The main purpose of the flutter analysis in section 3 is to study qualitatively the characteristic behaviors of transonic flutter such as the dip of the flutter boundary, a considerable decrease of the flutter frequency, the peculiar behavior of the flutter speed with the variation of mass ratio etc., which have been observed in the model tests\textsuperscript{19}.

2. DEVELOPMENT OF TRANSONIC SMALL PERTURBATION CODE

In this section, the outline of the present code based on the transonic small perturbation equation is described.

2.1. Basic Equations

A complete form of the transonic small perturbation equation is given by

$$
1-M_\infty^2-(r+1)M_\infty^{r\omega}\phi_x\phi_x + \phi_yy
-2M_\infty^{r\omega}\phi_x - M_\infty^{r\omega}\phi_t = 0
$$

(2-1)

In the above equation, there is some freedom to determine the value of $m$. In the present code $m$ is adjusted to match the critical pressure coefficient $C_{p^*}$ to its exact isentropic value\textsuperscript{19}.

For an airfoil oscillating with a small amplitude about a small mean angle of attack near the $y=0$ plane, the flow tangency condition can be given by

$$
\phi_y = f_1^{(i)}(x, t) + f_2^{(i)}(x) + f_2^{(i)}(x, t)
$$

on $y = \pm 0$ (2-2)

where $f_1^{(i)}(x)$ is the $y$-coordinate of the time mean contour of the airfoil and $f_2^{(i)}(x, t)$ is the unsteady displacement of the airfoil surface from the time mean contour.

The pressure coefficient $C_p$ can be calculated by the following relation:

$$
C_p = -2\phi_t - 2\phi_x
$$

(2-3)

The Kutta and the wake boundary conditions ($dC_p=0$ across the wake) are derived from Eq. (2-3) as:

$$
\phi_1 + \phi_2 = 0 \quad \text{on} \quad y=0, \quad x \geq 1
$$

(2-4)

where $d\phi$ is the potential jump across the wake.

At far-field boundaries, the following conditions are satisfied:

$$
\phi = \frac{\Gamma^{(i)}}{2\pi} \left\{ \frac{\pi}{2} \text{sgn}(y) + \tan^{-1} \left( \frac{\pi}{\beta_y} \right) \right\}
$$

for $M_\infty < 1$ (2-5)

which can be obtained as a far-field solution of the doublet distributed on the airfoil. In Eq. (2-5), $\Gamma^{(i)}$ is the mean steady state value of the circulation.
\[ \begin{align*}
\phi = 0 & \quad \text{upstream of the shock wave} \\
\phi_y = 0 & \quad \text{downstream of the shock wave}
\end{align*} \]
for \( M_\infty \geq 1 \)  

In the above, all the physical quantities are nondimensionalized by the semichord \( b \) and the freestream velocity \( U_\infty \).

2-2 Numerical Method

(a) Grid system

To obtain the smoothly stretching distribution of the grid points, a sufficiently large domain of the physical \( x-y \) plane is transformed into a computational plane \( \xi-\eta \) with constant mesh system. A special care must be taken, however, in selecting the stretching functions, especially for unsteady calculations. Bennett and Bland noticed the spurious oscillation in the variation of the aerodynamic coefficients with respect to the reduced frequency when they performed the calculations using the full potential code developed by Isogai, which employed the Cartesian grid system with the tangent stretching functions. They found that this spurious oscillation of the aerodynamic coefficients versus reduced frequency is a strong function of \( y \)-grid. It is conjectured that the probable cause of this phenomenon might be a hard stretching of the grid point distribution resulting from using the tangent stretching functions. In view of this, we introduced a stretching using polynomial functions (as shown later) which distributes more grid points in the intermediate and far-field in the physical plane than the tangent functions. As will be shown in the numerical examples, this remedy improved the results considerably, and, in fact, the spurious oscillations in the aerodynamic coefficient are completely eliminated for the range of the reduced frequency \( 0 \leq k \leq 0.5 \). Although some moderate spurious oscillations are still observed for \( k > 0.5 \), the reliable calculations of the aerodynamic coefficients for the range of the reduced frequency \( 0 \leq k \leq 0.5 \), might be enough for flutter analysis purposes.

The polynomial functions used as the stretching functions are

\[ y = C_1 \xi + C_2 \xi^2 \]
for \(-1 \leq \xi \leq 1\)  

where the constants \( C_1 \) and \( C_2 \) are given by

\[ C_1 = J y_{\min} (JZ - 1)/2, \quad C_2 = y_{\max} - J y_{\min} (JZ - 1)/2 \]

where \( J y_{\min} \) is the grid size at \( y = 0 \) plane and \( y_{\max} \) is the absolute value of the \( y \)-coordinate of the far-field boundary in the physical plane.

\[ x = -x_1 + C_3 (\xi + 1)^3 + C_4 (\xi + 1)^2 \]
for \(-3 \leq \xi \leq -1\)  

\[ x = 2(x_1 - C_4) \frac{\xi}{\xi^2 + 1} + C_5 \xi \]
for \(-1 \leq \xi \leq 1\)  

\[ x = x_1 + C_4 (\xi - 1)^3 + C_5 (\xi - 1)^2 \]
for \(1 \leq \xi \leq 3\)

where

\[ C_3 = \Delta x_{\min} (JZ - 1)/6.0, \quad C_4 = (x_{\max} - x_1 - 2C_3)/32.0 \]

where \( \phi \) determines the locations of the grid points adjacent to the leading and the trailing edges, and where \( \Delta x_{\min} \) is the grid size at the leading and the trailing edges, \( x_{\max} \) is the absolute value of the \( x \) coordinates of the upstream and the downstream boundaries in the physical plane.

Using these stretching functions, the potential equation (Eq. (2.1)) can be expressed in terms of \( \xi \) and \( \eta \) as

\[ (1 - M_\infty^2 - M_\infty^2 (\xi + 1) f(\phi_\xi) + g(\eta_\xi)) \]
\[- 2M_\infty f(\phi_\xi) - M_\infty \phi_\xi = 0 \]

where

\[ f = d\xi/dx, \quad g = d\eta/dy \]

The flow tangency condition (Eq. (2.2)) becomes

\[ g(\phi_\xi) - f(\phi_\xi) (x, t) + f(\phi_\xi) (x, t) + f(\phi_\xi) (x, t) \]
\[ = 0 \]

on \(-1 \leq \xi \leq 1 \) and \( \eta = \pm 0 \)  

Eq. (2.3) becomes

\[ C_p = -2\phi_{\xi} - 2\phi_{\xi} \]
\[ \text{on } \eta = \pm 0 \]

The Kutta and the wake boundary conditions become

\[ \phi_{\xi} + f \phi_{\xi} = 0 \]
\[ \text{on } \eta = \pm 1, \xi > 1 \]

(b) Finite-Difference Scheme

The finite difference scheme employed in the present code is a semi-implicit and implicit two-sweep procedure.

The finite difference form of Eq. (2.11) in the first semi-implicit sweep is

\[ K_{i,j} \phi_{\xi}(i_{i-1},j_{j-1}) + \Delta \frac{\partial F}{\partial \xi} \]
\[ + \frac{1}{2} (g(\phi) (g_{i_{i-1},j_{j-1}}) + g_{i_{i-1},j_{j-1}}) \]
\[- 2M_\infty f(\phi_{\xi}) (\phi_{\xi}) - M_\infty \phi_{\xi} + (\phi_{\xi}) = 0 \]

where the difference operators \( \phi_{\xi}, \phi_{\eta} \) etc. are defined by:

\[ \phi_{\xi} = ((y_{\eta} - y_{\eta})/\Delta \xi) \]
\[ \phi_{\eta} = ((x_{\xi} - x_{\xi})/\Delta \eta) \]

and where \( K_{i,j} \) is defined by
\[ K_{i,j} = 1 - M_{i,j}^2 - M_{\infty}^2 (r+1) f_{i,j} \delta t \phi_{i,j} \]

where \( \delta t \) is the centered difference operator.

Following the idea of the quasi-conservative scheme, a numerical viscosity term in divergence form \( \Delta \xi (\partial F^n / \partial \xi) \) is added in Eq. (2-16) to maintain numerical stability in the local supersonic region. The detailed form of \( \Delta \xi (\partial F^n / \partial \xi) \) is given by

\[ \Delta \xi \frac{\partial F^n}{\partial \xi} = F_{i,j} - F_{i-1,j} \]

(2-17)

where \( F_{i,j} \) is defined by

\[ F_{i,j} = \mu_{i,j} \frac{\partial F^n}{\partial \xi} \]

where the switching function \( \mu \) is defined by

\[ \mu_{i,j} = \max \{ 0, (r+1) M_{\infty}^2 f_{i,j} \phi_{i,j} + M_{\infty}^2 - 1 \} \]

It should be noted that the addition of this numerical viscosity term (Eq. (2-17)) generates Murman's four operators.

The equations (2-16) form a tri-diagonal system of equations to be solved for \( \phi_{i,j} \) on column \( i \). It should be noted that the one sweep from the upstream to downstream is enough to obtain the \( \phi_{i,j} \) because there is no unknowns downstream of column \( i \) at the time of tri-diagonal inversion.

For the second sweep at the same time level, the following difference expression is employed:

\[ K_{i,j} \delta t \frac{\partial F_{n+1}^n}{\partial \xi} + \Delta \xi \frac{\partial F_{n+1}^n}{\partial \xi} + g_{f_{i,j}} \left( g_{f_{i,j}} \frac{\partial \phi_{i,j}^{n+1}}{\partial \xi} \right) - 2 M_{\infty}^2 J_{i,j} (\delta t) \phi_{i,j}^{n+1} - M_{\infty}^2 \delta t (\delta t) \phi_{i,j}^{n+1} = 0 \]

(2-18)

Therefore the second sweep is fully implicit, but no simultaneous algebraic solutions nor relaxation of the tri-diagonal equations is necessary for solving Eqs. (2-18) since the neighboring values are already known from the first semi-implicit sweep and \( \phi_{i,j}^{n+1} \) can be eliminated by a simple point inversion.

Although this two-sweep scheme is still conditionally stable, the addition of the second implicit sweep increases the allowable time step size considerably over the scheme which uses only the first semi-implicit sweep.

(c) Boundary Conditions

In order to satisfy the flow tangency condition (Eq. (2-13)) on \( \eta = \pm 0 \) plane \((-1 \leq \xi \leq 1)\), \( g_{\phi_0} \) is approximated by the following second order-accurate finite difference form. For the upper surface, we obtain

\[ g_{\phi_0} = \frac{2 f_{ij}^2}{\Delta \xi} (-3 \phi_{ij}^{n+1} + 4 \phi_{ij}^{n+1} - \phi_{ij}^{n+1}) \]

(2-19)

where \( f_{ij} \) is the grid point index at \( \eta = 0 \). Substituting Eq. (2-19) into Eq. (2-13), we obtain

\[ \phi_{ij}^{n+1} = \frac{4}{3} \phi_{ij}^{n+1} - \frac{1}{3} \phi_{ij}^{n+1} - \frac{2 \Delta \xi}{3 g_{f_{ij}}} \left( f_{i,j}^{(1)}(x, t) + f_{i,j}^{(2)}(x, t) \right) \]

(2-20)

Although the unknowns \( \phi_{ij}^{n+1} \) and \( \phi_{ij}^{n+1} \) are included in the R.H.S. of Eq. (2-20), these can be directly incorporated into the tri-diagonal equations derived from Eq. (2-16). The similar expression of \( \phi_{ij}^{n+1} \) for the lower surface of \( \eta = 0 \) plane can be easily obtained.

The potential jump across the wake (\( \eta = 0, \xi > 1 \)) is calculated from Eq. (2-15) expressed in the following finite difference form:

\[ \Delta \phi_{i,j}^{n+1} = \left( \phi_{i,j}^{n+1} + f_{i-1,j} \left( \frac{\Delta t}{\Delta \xi} \right) \Delta \phi_{i-1,j}^{n+1} \right) / \left( 1 + f_{i-1,j} \left( \frac{\Delta t}{\Delta \xi} \right) \right) \]

(2-21)

Once the value of \( \Delta \phi_{i,j}^{n+1} \) at the trailing edge is determined, all the \( \Delta \phi_{i,j}^{n+1} \) in the wake region can be calculated from Eq. (2-21).

The numerical experiments were performed to determine the appropriate values of the constants associated with the grid point distribution functions Eqs. (2-7)-(2-10). The values thus determined are \( x_1 = 0.08, \Delta x_{\min} = 0.04, x_{\max} = 0.40 \) and \( y_{\max} = 120.0 \). The allowable time step was also determined by the numerical experiments and the value of \( \Delta t = 0.08 \) was used for the most of the calculations reported here. The number of grid points used are 91 (in \( x \) direction) by 81 (in \( y \) direction), unless otherwise noted.

The unsteady calculations have been started with the mean steady state solutions as initial values, which have been obtained as a limit of the unsteady calculations by using the present code with the steady state boundary conditions. To obtain the periodic solutions, it takes about 3-10 cycles of oscillation depending on the reduced frequency and Mach number. Harmonic analyses of the time varying pressure and load distributions, total lift and pitching moment were made. As is discussed in Section 3, the first harmonic components (in-phase and out-of-phase components) are the most important for the determination of the flutter boundary. The computation time is presently 490 sec per oscillatory cycle for \( k = 0.10 \) on FACOM 230-75 vector processor.
2-3 Evaluation of the Code

In this subsection, the validity of the code is checked in various ways, namely, comparison with the linear theory for a flat-plate case, comparison with Ballhaus and Steger's fully conservative, low-frequency small perturbation results\(^{19}\), and comparison with the Davis and Malcolm's experimental data\(^{20}\) for a NACA64A010 airfoil. The capability of the present code for calculating the case of the supersonic freestream Mach numbers is also demonstrated.

In Figs. 1a and 1b, the variations of the total lift and pitching moment coefficients versus reduced frequency for a flat-plate oscillating in pitch about midchord at Mach number 0.75 are compared with the linear theory (doublet lattice method\(^{21}\)). The agreement of the present finite difference calculations with those of the linear theory seems to be satisfactory. The small difference between the two results are probably attributable to the loss of the leading edge suction for the finite difference calculation which is based on the nonlinear equation. In Figs. 2a and 2b, the load distributions calculated by the present finite difference method and the linear theory for an oscillating flat-plate at Mach number 0.75 and \(k=0.15\) and \(k=0.50\) are also compared, respectively. The agreement between the two is excellent.

In order to see whether the present quasi-conservative scheme can capture the shock wave motions correctly, the calculations are performed for a thickening and thinning circular-arc airfoil problem for which the results based on the low-frequency, fully conservative scheme are available\(^{22}\). The variation of the thickness to chord ratio \(\tau\) with respect to time \(t\) is defined by

\[
\tau = \begin{cases} 
0.10(10-15(t/30)+6(t/30)^2)(t/30)^3 & 0 \leq t \leq 30 \\
0.10(10-15(2-t/30)+6(2-t/30)^2)(2-t/30)^3 & 30 \leq t \leq 60 
\end{cases}
\]

The Mach number considered is 0.85. As shown in Fig. 3, the shock motions predicted by the present code\(^*\) agree well with those predicted by Ballhaus and Steger\(^{23}\).

Another check of the present code can be made by the comparison with the experimental data. For this purpose, the calculations are performed for a NACA64A010 airfoil oscillating in pitch about quarter-chord axis at Mach

\* 181x121 grid points (61 points on the airfoil) are used for this thickening and thinning airfoil problem.

Fig. 1a. Lift coefficients versus reduced frequency for a flat-plate oscillating in pitch about midchord.

Fig. 1b. Pitching moment coefficients (around midchord axis, positive nose up) versus reduced frequency for a flat-plate oscillating in pitch about midchord.
Fig. 2a. Unsteady load distributions for a flat-plate oscillating in pitch about midchord, $M_\infty = 0.75$, $k = 0.15$.

Fig. 2b. Unsteady load distributions for a flat-plate oscillating in pitch about midchord, $M_\infty = 0.75$, $k = 0.50$.

Fig. 3. Unsteady pressure distributions on thickening-thinning circular-arc airfoil.

number 0.80, for which the experimental data of Davis and Malcolm are available. In Fig. 4, the mean steady pressure distribution is shown, being compared with the experimental data. A relatively weak shock is present at the midchord. In Fig. 5, the chordwise distributions of the in-phase and out-of-phase components of the first harmonic of the upper surface pressure are shown, being compared with the experimental data. The Mach number and the reduced frequency considered are 0.8 and 0.20, respectively. The peak value of the in-phase component and the rapid variation of
Fig. 4. Steady pressure distribution on NACA64A010 airfoil.

Fig. 5. Upper surface unsteady pressure distributions on NACA64A010 airfoil oscillating in pitch about quarter-chord.

Fig. 6a. Theoretical and experimental lift versus reduced frequency for NACA64A010 airfoil oscillating in pitch about quarter-chord.

Fig. 6b. Theoretical and experimental pitching moment (about leading edge, positive nose up) versus reduced frequency for NACA64A010 airfoil oscillating in pitch about quarter-chord.
the out-of-phase component seen around mid-chord are the effects of the shock wave. The agreement of the present finite difference calculations with the experimental data is satisfactory. In Figs. 6a and 6b, the variations of the in-phase and out-of-phase components of the first harmonic of lift and pitching moment (about the leading edge) versus reduced frequency are compared with the experimental data and those predicted by the linear theory (doublet lattice method**). The agreement between the present calculations (TSP code) and the experimental data is good especially for the out-of-phase components of both lift and pitching moment for the range of the reduced frequency $0.10 \leq k \leq 0.30$. The difference between the two (TSP code and the experiments), however, becomes larger as the reduced frequency getting smaller than 0.10. This might be attributed to the effects of the shock wave and turbulent boundary layer interactions, which are neglected in the present potential flow calculations.

In order to show the capability of the present code for treating the cases of higher transonic Mach number range, the steady and unsteady flow fields around a six percent thick parabolic-arc airfoil at Mach number 1.15 have been calculated. In Fig. 7, the mean steady pressure distribution on $y=0$ plane is plotted. The airfoil is located between $-1 \leq x \leq 1$. As seen from the figure, there are two shock waves; one is the detached shock wave at the distance of about 0.09 chord away from the leading edge and the other is the supersonic to transonic shock wave at the trailing edge. It was confirmed that the position of the detached shock wave, which is predicted by the present code, shows a close agreement with that of Murman’s fully conservative relaxation calculation reported in Ref. 25. The in-phase and out-of-phase components (first harmonics) of the unsteady load distributions on this airfoil oscillating in pitch about midchord about zero mean angle of attack with the amplitude of 0.1 degrees at the same Mach number, are shown in Fig. 8. The reduced frequency considered is $k=0.15$.

![Fig. 7. Steady pressure distribution on 6 percent thick parabolic-arc airfoil, $M_\infty=1.15$, $\alpha=0^\circ$.](image-url)
6-PERCENT THICK PARABOLIC-ARC AIRFOIL
\[ M_x = 1.15, \quad \alpha = 0'\]
\[ \alpha = 0' \pm 0.1' \sin kt \]
\[ k = 0.15 \]

Fig. 8. Unsteady load distributions (in-phase and out-of-phase components of the first harmonics) on 6 percent thick parabolic-arc airfoil oscillating in pitch about midchord, \( M_w = 1.15, k = 0.15 \).

3. FLUTTER ANALYSIS OF NACA-64A010 AIRFOIL WITH TWO-DEGREES-OF-FREEDOM

In this section, the unsteady aerodynamic forces calculated by the small perturbation code developed in section 2 are applied to the flutter analysis of a NACA64A010 airfoil with two-degrees-of-freedom.

3-1. Basic Equations and Solution Procedure

A sketch of the binary system treated here is shown in Fig. 9. The equations of motion of the system can be given by
\[ m\ddot{h} + S_o\dot{h} + m\omega_n^2 h = -L \]  
(3-1)
\[ S_o\dot{h} + I_o\dot{\alpha} + I_o\omega_o^2 \alpha = M_y \]  
(3-2)
where \( L \) and \( M_y \) are the total lift (positive up) and pitching moment (about elastic axis, positive nose up) per unit span and can be expressed, in general, by the following Fourier series form:

\[ L/(\frac{1}{2}\rho_w U_w^2 C) = a_{L,0}/2 + \sum_{n}^{N} (a_{L,n} \cos (nt) + b_{L,n} \sin (nt)) \]
(3-3)
\[ M_y/(\frac{1}{2}\rho_w U_w^2 C) = a_{M,0}/2 + \sum_{n}^{N} (a_{M,n} \cos (nt) + b_{M,n} \sin (nt)) \]
(3-4)
where the coefficients \( a_{L,n} \) and \( a_{M,n} \) (\( n = 0, 1, 2, \ldots \)) are generally nonlinear functions of the amplitude of oscillation of airfoil because the basic equation (TSP equation) is nonlinear. It is also possible to express the responses of the airfoil (\( h \) and \( \alpha \)) in the following Fourier series:
\[ h = h_0/2 + \sum_{n}^{N} \left[ h_{L,n} \cos (nt) + h_{R,n} \sin (nt) \right] \]
(3-5)
\[ \alpha = \alpha_0/2 + \sum_{n}^{N} \left[ \alpha_{L,n} \cos (nt) + \alpha_{R,n} \sin (nt) \right] \]
(3-6)

Substituting Eqs. (3-3)–(3-6) into Eqs. (3-1) and (3-2), and equating the first harmonic components in the both sides of Eqs. (3-1) and (3-2), we obtain the following equations (expressed in complex form):
\[-m\omega^2 h_1 - S_o\omega^2 \alpha_1 + \omega_n \omega^2 \bar{h} = -\frac{1}{2}\rho_w U_w^2 C \cdot C_{L,1} \]
(3-7)
\[-S_o\omega^2 \alpha_1 + I_o\omega_o^2 \alpha_1 = \frac{1}{2}\rho_w U_w^2 C \cdot C_{M,1} \]
(3-8)
where \( h_1, \alpha_1, C_{L,1}, \) and \( C_{M,1} \) are defined by
\[ h_1 = h_{R,1} + ih_{I,1} \]
\[ \alpha_1 = \alpha_{R,1} + i\alpha_{I,1} \]
\[ C_{L,1} = b_{L,1} + ia_{L,1} \]
\[ C_{M,1} = b_{M,1} + ia_{M,1} \]
(3-9)
where \( i \) is defined by \( i = \sqrt{-1} \).

As long as we are concerned about the determination of the flutter boundary, it is only needed to treat a small amplitude of oscilla-
Fig. 10a. Effect of amplitude of airfoil oscillation on unsteady lift coefficient (first harmonic components).

Fig. 10b. Effect of amplitude of airfoil oscillation on unsteady pitching moment coefficient (first harmonic components, about midchord axis, positive nose up).

tion. In that case, the numerical experiments show that the higher harmonic components in the total lift and pitching moment are very small compared with the first one. Therefore, the flutter boundary can well be determined from Eqs. (3-7) and (3-8), which are consisted of the first harmonic components. In order that the conventional $U$-$\alpha$ method of flutter analysis can be applied to the present problem, the linear dependence of $C_{L,1}$ and $C_{M,1}$ on $h_1$ and $\alpha_1$ must be confirmed. In Figs. 10a–10b, the linear dependence of $C_{L,1}$ and $C_{M,1}$ on the amplitude of oscillation of the airfoil is examined. The Mach numbers considered are 0.80 and 0.85 respectively. A relatively weak shock is present about midchord at $M_\infty = 0.80$ and the strong shock wave is present around $3/4$ chord position at $M_\infty = 0.85$. As seen in the figures, the $C_{L,1}$ and $C_{M,1}$ (both real part and imaginary part) vary linearly as long as the amplitude of oscillation remains small. It should be noted that this linearity of $C_{L,1}$ and $C_{M,1}$ with respect to the amplitude of oscillation also guarantees the modal superposition principle of $C_{L,1}$ and $C_{M,1}$, namely, we can predict the $C_{L,1}$ and $C_{M,1}$ for an arbitrary mode of oscillation by combining $C_{L,1}$ and $C_{M,1}$ of two independent known modes. Such linearity and the superposition principle of the aerodynamic forces in transonic regime have also been confirmed experimentally by Davis and Malcolm for the same airfoil of NACA64A-010 at $M_\infty = 0.80$.

All the aerodynamic coefficients used for the present flutter analysis are calculated with the amplitude of oscillation of 0.1 degrees (or less) and its equivalence for a pitching and a plunging mode of oscillation, respectively.

*
Thus the $C_{L1}$ and $C_{M1}$ in the rights of Eqs. (3-7) and (3-8) can be written as

$$C_{L1} = -\pi k^2 \left( \frac{h_1}{b} \right) L_1 + \alpha_1 M_2$$

$$C_{M1} = \frac{1}{2} \pi k^2 \left( \frac{h_1}{b} \right) M_1 + \alpha_1 M_2$$

(3-10)

(3-11)

where $L_1$ and $M_1$, and $L_2$ and $M_2$ are the lift and pitching moment coefficients (in complex form) obtained for a plunging and a pitching mode of oscillation respectively, and these coefficients are independent of $h_1$ and $\alpha_1$. Substituting Eqs. (3-10) and (3-11) into Eqs. (3-7)-(3-8), and introducing the structural damping $g$, we obtain the well-known flutter determinant of the binary system as:

$$\rho (1 - R^2 Z) + L_1 \rho (x_c g - \alpha) + L_2 = 0$$

$$\rho (x_c g - \alpha) + M_1 + \mu r_s (1 - Z) + M_2 = 0$$

(3-12)

where $Z$ is defined by

$$Z = \left( \frac{\omega_1}{\omega_2} \right)^2 (1 + ig)$$

Since the procedure for determining the flutter speed and frequency from Eq. (3-12) are well established ($U-g$ method), we will not mention about it.

The aerodynamic coefficients $L_1$, $L_2$, $M_1$, $M_2$ related to an arbitrary elastic axis position $a$ can easily be obtained from those calculated at some particular axis $a'$ by the following simple transformation:

$$
\begin{pmatrix}
L_1 \\
L_2 \\
M_1 \\
M_2
\end{pmatrix} =
\begin{pmatrix}
1 & 0 & 0 & 0 \\
\alpha' - a & 1 & 0 & 0 \\
\alpha' - a & 1 & 0 & 0 \\
(\alpha' - a)^2 & \alpha' - a & \alpha' - a & 1
\end{pmatrix}
\begin{pmatrix}
L_{1}' \\
L_{2}' \\
M_{1}' \\
M_{2}'
\end{pmatrix}
$$

(3-13)

In this paper, the aerodynamic coefficients are evaluated with reference to the midcord axis by using the TSP code developed in Sec. 2. The reduced frequencies, for which the calculations by the TSP code were performed, are $k = 0.05$, 0.15 and 0.30 for Mach numbers at 0.70, 0.75, 0.775, 0.80, 0.90 and 1.01, respectively, and are $k = 0.05$, 0.10, 0.15, 0.20, 0.30 and 0.50 for Mach numbers at 0.825 and 0.85, respectively. The aerodynamic coefficients at a reduced frequency between the above values are estimated by using the cubic spline interpolation.

3-2 Behavior of Steady and Unsteady Aerodynamic Forces Acting on NACA64A010 Airfoil

Before we present the results of the flutter analyses, some remarks are given in this sec-

-Continued.
Mach number is slightly below $M_\infty = 0.775$. With increasing Mach number, the weak shock wave is formed around midchord at $M_\infty = 0.80$. When the Mach number is further increased, the shock wave moves downstream with increasing strength. At $M_\infty = 0.85$, the shock is located at about $3/4$ chord position, and at $M_\infty = 0.90$, the shock reaches the trailing edge and the fishtail shock pattern appears. At $M_\infty = 1.01$, there appears a detached shock wave at far-upstream of the airfoil and a supersonic to supersonic shock wave is present at the trailing edge. The unsteady load distributions corresponding to these Mach numbers are shown in Figs. 12a–12h, respectively. The airfoil is oscillated in pitch about the midchord axis with the amplitude of 0.1 degrees. The reduced frequency considered is 0.15. In the figures, the in-phase and out-of-phase components of the first harmonic of the pressure dif-

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Fig. 11c. $M_\infty = 0.775$
---Continued.

Fig. 11d. $M_\infty = 0.80$
---Continued.

Fig. 11e. $M_\infty = 0.825$
---Continued.

Fig. 11f. $M_\infty = 0.85$
---Continued.
ferences are plotted*. From the load distributions at $M_\infty = 0.70 - 0.775$ (see Figs. 12a-12c), we can clearly see how the load distribution at $M_\infty = 0.70$, which shows the pattern similar to that predicted by the linear theory**, becomes distorted by the nonlinear compressibility effects with increasing Mach number. When Mach number is further increased from $M_\infty = 0.775$, there appear the peak values in the load distributions, which are the effects of the shock wave. The peak values in the out-of-phase components at $M_\infty = 0.825$ and $M_\infty = 0.85$ are especially large. This means that there is a considerable amount of phase lag in the motions of the shock wave at these Mach num-

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* At $M_\infty = 0.875$, it was not possible to obtain the periodic solution though the solution is numerically stable. The reason for this is not known yet.

---

** At $M_\infty = 0.875$, it was not possible to obtain the periodic solution though the solution is numerically stable. The reason for this is not known yet.
Fig. 12a. $M_a=0.70$ Unsteady load distributions (in-phase and out-of-phase components of first harmonics) on NACA64A010 airfoil oscillating in pitch about midchord.

Fig. 12b. $M_a=0.75$ —Continued.

Fig. 12c. $M_a=0.775$ —Continued.
Fig. 12e. $M_\infty = 0.825$
—Continued.

Fig. 12f. $M_\infty = 0.85$
—Continued.

Fig. 12g. $M_\infty = 0.90$
—Continued.

Fig. 12h. $M_\infty = 1.01$
—Continued.
bers. It should also be noted that the magnitudes of the pressure difference (both in-phase and out-of-phase components) upstream of the shock wave gradually decrease with increasing Mach number from \( M_\infty = 0.80 \) to \( M_\infty = 0.85 \). This phenomenon corresponds with the expansion of the local supersonic region upstream of the shock as Mach number is increased. Since the shock wave has already reached the trailing edge at \( M_\infty = 0.90 \), there is no peak values (both in-phase and out-of-phase components) in the load distributions at \( M_\infty = 0.90 \) and \( M_\infty = 1.01 \) (Figs. 12g and 12h). To see the effects of the reduced frequency on the load distributions, the in-phase and out-of-phase components of the first-harmonics of the pressure differences at \( M_\infty = 0.80 \) are plotted with the reduced frequency as a parameter in Fig. 13. A considerable effect of the reduced frequency can be seen especially for the in-phase

Fig. 13. Effects of reduced frequency on unsteady load distributions (in-phase and out-of-phase components of first harmonics) on NACA64A010 airfoil oscillating in pitch about midchord axis, \( M_\infty = 0.80 \).

Fig. 14a. Unsteady lift coefficients (in-phase and out-of-phase components of first harmonics) versus Mach number for NACA64A010 airfoil oscillating in pitch about midchord.

Fig. 14b. Unsteady pitching moment coefficients (in-phase and out-of-phase components of first harmonics) versus Mach number for NACA64A010 airfoil oscillating in pitch about midchord axis.
component. In order to see the behavior of the total lift and pitching moment with respect to the variation of Mach number and the reduced frequency, the in-phase and out-of-phase components of the first harmonics of the total lift and pitching moment versus Mach number are plotted with the reduced frequency as a parameter in Figs. 14a and 14b, respectively. The reduced frequencies considered are 0.05, 0.15 and 0.30. The notable points in these figures are that the magnitudes of the in-phase and out-of-phase components of the lift decrease rapidly with increasing reduced frequency especially at Mach numbers 0.825 and 0.85, and the rapid variation of the pitching moment in the range of Mach number 0.80–0.90, which is caused by the movement of the mean position of the shock wave. To see the behavior of the lift and pitching moment versus reduced frequency in detail, the variations of the in-phase and out-of-phase components of the first harmonics of lift and pitching moment versus reduced frequency at \( M_{\infty} = 0.85 \), for which the strong shock is present at 3/4 chord position, are shown in Figs. 15a and 15b, respectively. For comparison, the values predicted by the linear theory are also plotted in the same figures.

3-3 Results of Flutter Analysis

Flutter calculations are performed for two typical binary systems; one of which simulates the vibrational characteristics of a typical chordwise section of a sweptback wing, and the other of which simulates that of an unswept wing. We will call the former case as Case A and the latter as Case B.

3-3.1 Results for Case A

Giving the following values to the structural parameters of the binary system shown

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**Fig. 15a.** Unsteady lift coefficients (in-phase and out-of-phase components of first harmonics) versus reduced frequency for NACA64A010 airfoil oscillating in pitch about midchord axis, \( M_{\infty} = 0.85 \).

**Fig. 15b.** Unsteady pitching moment coefficients (about midchord axis, positive nose up, in-phase and out-of-phase components of first harmonics) versus reduced frequency for NACA64A010 airfoil oscillating in pitch about midchord axis, \( M_{\infty} = 0.85 \).
in Fig. 9,\n\[ a = -2.0, \; x_{c1} = -0.2, \; r_{c1} = 0.24, \; \omega_\mu / \omega_\alpha = 1.0 \]
we obtain the system having the natural modes shown in Fig. 16a. This system is identical to
that used in Ref. 30 for the investigation of the transonic dip mechanism of a sweptback
wing flutter. By employing the linear aerodynamic theory (doublet lattice method) in the
flutter calculations, it is concluded in Ref. 30 that the mechanism of the single-degree-of-freedom flutter is dominating at the bottom of the transonic dip. This conclusion reached in
Ref. 30 is more definitively confirmed in the present analysis using the nonlinear aerodynamic forces predicted by the TSP code as will be discussed later.

In Figs. 16a and 16b, the calculated flutter velocity coefficients and flutter frequencies of this system for \( \mu = 60 \) are plotted, respectively, as a function of Mach number, being compared with those predicted by using the linear aerodynamic theory (doublet lattice method)\(^*\).

As shown in Fig. 16a, a sharp transonic dip of the flutter boundary is predicted when the nonlinear aerodynamic forces (TSP code) are used. When we look at the steady-state pressure distributions and the corresponding unsteady load distributions already shown in Sec. 3-2, we first notice that the behavior of the shock wave versus Mach number has a close relation with the behavior of the flutter boundary. At Mach numbers 0.825 and 0.85, when the flutter speed takes a minimum value, the shock wave is located at 60 percent chord position at \( M = 0.825 \) and 75 percent chord position at \( M = 0.85 \), respectively, on the airfoil (see Figs. 11e–11f). However, at \( M = 0.90 \), when the flutter speed takes as high as six times of that at \( M = 0.825 \) and \( M = 0.85 \), the shock wave has reached the trailing edge (see Fig. 11g). Such differences of the mean steady state shock patterns are well reflected in the unsteady load distributions shown in Figs. 12e–12g. In Fig. 17, the unsteady load distributions at \( M = 0.825 \) and \( M = 0.90 \) are plotted in the same figure to make the comparison easier and to contrast the effects of the shock wave. At \( M = 0.825 \), there is a sharp peak value at about 60 percent chord position in the

\(^*\) As already mentioned in the subsection 3-2, the periodic solution of the unsteady aerodynamic forces was not obtained at \( M = 0.875 \). Therefore, the flutter points at \( M = 0.85 \) and \( M = 0.90 \) are connected by the dotted line.

Fig. 16a. Natural vibration modes, and flutter velocity coefficient versus Mach number for NACA64A010 airfoil at zero mean angle of attack (Case A: \( a = -2.0, \; x_{c1} = -0.2, \; r_{c1} = 0.24, \; \omega_\mu / \omega_\alpha = 1.0 \)).

Fig. 16b. Flutter frequency versus Mach number for NACA64A010 airfoil at zero mean angle of attack (Case A).
out-of-phase component, while at $M_{\infty} = 0.90$, there is no such peak value in both the in-phase and out-of-phase components because the shock is already at the trailing edge. This difference between the load distributions at $M_{\infty} = 0.825$ and $M_{\infty} = 0.90$ is especially important when we consider the possible cause of the large difference of the flutter speeds between the two. It can be easily confirmed that the large negative value of the out-of-phase component produces large negative damping for the first natural mode of this system, for which the pivotal point is located about 1.5 chord ($x_P = -3.87$) upstream of the leading edge. Thus, the large negative peak value of the out-of-phase component at 60 percent chord position for $M_{\infty} = 0.825$ enables the occurrence of a single-degree-of-freedom flutter whose mode is almost identical to the first natural mode. This can be seen by examining the flutter modes shown in Fig. 18, where the amplitude ratio and the phase difference between the plunging and pitching motions at the elastic axis versus mass ratio are plotted for three typical Mach numbers. At $M_{\infty} = 0.825$, the phase difference rapidly decreases to zero with increasing mass ratio, while the amplitude ratio approaches to the constant value of 1.86, when $\mu$ is greater than 40. Since the zero phase difference between the plunging and the pitching motion implies the existence of the pivotal point at $x_P = a - (h/b)/\alpha$, the flutter mode at $M_{\infty} = 0.825$ for $\mu$ greater than 40 is essentially a pitching oscillation with pivotal point at $x_P = -3.86$, which is almost identical to the first natural mode shown in Fig. 16a. Therefore when we recall the fact that the large negative peak value of the out-of-phase component in the load distribution at $M_{\infty} = 0.825$ is due to the phase lag of the shock wave.

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**Fig. 17.** Comparison of unsteady load distributions at $M_{\infty} = 0.825$ and $M_{\infty} = 0.90$ for NACA64A010 airfoil oscillating in pitch about midchord axis.

**Fig. 18.** Flutter modes (amplitude ratio and phase difference) versus mass ratio for Case A.
motion, it is concluded that the shock wave (especially the phase delay of the shock wave) is playing the dominant role in the mechanism of the transonic dip phenomenon. The importance of the role of the shock wave in the transonic flutter of a wing is also pointed out by Ashley in Ref. 31.

Another point of interest in the present flutter analysis is the behavior of the flutter velocity and frequency versus mass ratio. In Figs. 19a and 19b, the dimensionless flutter velocity and frequency are plotted against mass ratio for three typical Mach numbers. The remarkable feature in Fig. 19a is the behavior of the flutter speed at $M_\infty=0.825$, when the flutter speed decreases with increasing mass ratio. This has a sharp contrast with the behavior of flutter speed at $M_\infty=0.70$ or $M_\infty=0.90$, when the flutter speed increases with increasing mass ratio, that is commonly experienced for usual bending-torsion flutter. This unusual behavior of flutter speed at $M_\infty=0.825$ is closely related with the mechanism of the flutter at this Mach number. When we look at the flutter mode shown in Fig. 18, we recognize that the flutter around $\mu=10$ is a classical type flutter since the phase difference $\phi_{h,n}$ is large, while the flutter for $\mu=\geq40$ is the single-degree-of-freedom flutter as already mentioned. Therefore the flutters in between the mass ratios $\mu=10$ and $\mu=40$, can be regarded as the transition from the classical type flutter to the single-degree-of-freedom flutter.

3.3.2 Results for Case B

The natural modes of the system for Case B are shown in Fig. 20a. The structural parameters used are as follows:

$a=-0.30$, $x_0=0.2$, $r_{st}s=0.24$, $\omega_n/\omega_s=0.20$

In Figs. 20a and 20b, the flutter velocity coefficient and flutter frequency for $\mu=60$ are plotted against Mach number, being compared with those predicted by using the linear aerodynamic forces (doublet lattice method). As seen from Fig. 20a, the transonic dip is mild in this case and the discrepancy between the TSP code calculation and the doublet lattice calculation is relatively small for $M_\infty\leq0.825$ and is quite large for $M_\infty\geq0.85$. In Fig. 21, the flutter modes, namely, the amplitude ratio and phase difference of the plunging and pitching motions at the elastic axis position, are plotted against mass ratio for three typical Mach numbers. Although all the flutters at these three Mach numbers are classical type (coupled bending and torsion) flutters since the phase differences remain finite for the range of the mass ratio considered, the rapid increase of the amplitude ratio and the decrease of the phase difference with increasing mass ratio at $M_\infty=0.825$ indicate that the mechanism of the single-degree-of-freedom flutter (bending mode

* Mykytow has reported that some model tests of sweptback wings show a significant increase in flutter dynamic pressure as mass ratio is further decreased from low value.
predominant) is also dominating the flutter at this Mach number when the mass ratio is large. In Figs. 22a and 22b, the dimensionless flutter velocity and frequency are plotted against the mass ratio for three typical Mach numbers.

Fig. 20a. Natural vibration modes, and flutter velocity coefficient versus Mach number for NACA64A010 airfoil at zero mean angle of attack (Case B: \( \alpha = -0.30, \; \gamma = 0.20, \; \gamma^2 = 0.24, \; \omega_n/\omega_m = 0.20 \)).

Fig. 20b. Flutter frequency versus Mach number for NACA64A010 airfoil at zero mean angle of attack (Case B).

Fig. 21. Flutter modes (amplitude ratio and phase difference) versus mass ratio for Case B.
4. CONCLUDING REMARKS

Transonic flutter characteristics of a NACA64A010 airfoil with two-degrees-of-freedom (plunging and pitching) are investigated theoretically. For this purpose a computer program based on the transonic small perturbation equation, which can be applied for the wide range of the reduced frequency \(0 \leq k \leq 0.50\) and Mach numbers (from subcritical to above Mach 1), has been developed. The validity of the code has been checked by comparing the results of the present code with those of linear theory for a flat-plate case and with those of Ballhaus and Steger's fully conservative, low-frequency small disturbance code for thickening and thinning circular-arc airfoil problem. A satisfactory agreement has been obtained in these comparisons. The unsteady pressure distribution, total lift and pitching moment on an oscillating NACA64A010 airfoil at Mach number 0.80 are also calculated by the present code, being compared with the Davis and Malcolm's experimental data. Agreement between the two is good for the pressure distributions and satisfactory for the total lift and pitching moment for the range of the reduced frequency \(0.10 \leq k \leq 0.30\).

Before the conventional \(U-g\) method is applied to the flutter analysis, the superposition principle of the aerodynamic forces has been checked and it has been confirmed that the superposition of the aerodynamic forces is guaranteed, even at \(M_\infty = 0.85\) when a strong shock is located on the airfoil, as long as the amplitude of the airfoil oscillation remains small. The flutter calculations are performed for two typical binary systems, one of which simulates the vibrational characteristics of a typical streamwise section of a sweptback wing, and the other of which simulates that of an unswept wing. For the former case, the sharp and deep transonic dip of the flutter boundary around \(M_\infty = 0.825\) has been predicted for the mass ratio \(\mu \geq 40\). By the examination of the flutter modes, it is disclosed that the mechanism of the single-degree-of-freedom flutter is dominating the flutter at the bottom of the transonic dip. It is concluded from the examination of the unsteady load distributions that the large negative damping produced by the phase lag of the shock wave motion is the cause of the occurrence of the single-degree-of-freedom (the first natural mode) flutter and hence, the cause of the transonic dip phenomenon. The peculiar behavior of the flutter speed versus mass ratio (the flutter speed increases significantly with decreasing the mass ratio from low value), which has been observed experimentally in the flutter tests of the sweptback wings, has also been predicted qualitatively by the present calculations. In contrast with the former case, the transonic dip of flutter boundary predicted for the latter case is relatively shallow, though the flutter...
mode at the bottom of the dip is still bending mode predominant.

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