# Simulations of Stratified Flows in Rotating System by Two Fluid Mixture Model of Lattice Boltzmann Method

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Abstract

Two-particle (blue and red) type lattice Boltzmann model is applied to simulate rotating stratified flows. Gravitational force acts on one kind of particles (blue ones) and the Coriolis force acts on both kinds of particles. The results of the natural convection in a rotating system support the varidity of this model, and three-dimensional gravity current and the rotating effect on it are also studied.

***Key words:***

*Lattice Boltzmann method, Stratified flow, Rotating flow, Natural convection, Gravity current*

## 1. Introduction

The lattice Boltzmann method can be considered one branch of the cellular-automata[1][2]. Actually, this method has been developed from the lattice gas automata[3][4], and the time-evolution equation governs this method, the lattice BGK equation, the state, that is the particle distribution function, of any lattice site is determined by the states on the neighboring lattice sites at every time step. This method is now a powerful tool of computational fluid dynamics and the readers can obtain information of the trend by reviews[3][4] or books[5]-[8]. In this paper, we adopted two-fluid miscible model to simulate three-dimensional stratified flows in a rotating system. In geo-physical fluid dynamics, the body forces (gravitational force and Coriolis force) and the density difference are essential factors. But the effect of the centrifugal force is rather small and is included in the gravity. The density difference is realized by changing the rate of the two components and the body force is introduced to the local equilibrium distribution f unctions of particles.

## 2. Lattice Boltzmann Model

The lattice Boltzmann method is based on the particle motions, and calculate the distribution function of the particles by the following lattice Bhatnager -Gross-Krook (BGK) equation

$$f_i(\mathbf{r}+\mathbf{c}\tau, t+\tau) = f_i(\mathbf{r}, t) - \frac{1}{\phi}\left\{f_i(\mathbf{r}, t) - f_i^{(0)}(\mathbf{r}, t)\right\} \quad (1)$$

where $f_i$ is the particle distribution function, $f_i^{(0)}$ on the RHS refers to its local equilibrium state, subscript $i$ represents particle velocity direction, $\mathbf{c}$ is the particle velocity, and $\phi$ is called the relaxation time factor. The term on RHS represents the collision. As mentioned above, two fluids (two kinds of particles, say red and blue ones) are employed

Macroscopic variables are obtained from the distribution function,

$$\rho_b = \sum_i f_{bi}, \quad \rho_r = \sum_i f_{ri} \quad (2)(3)$$

for the densities, and

$$(\rho_b + \rho_r)\mathbf{u} = \sum_i (f_{bi}\mathbf{c}_i + f_{ri}\mathbf{c}_i) \quad (4)$$

for the momentum, Subscripts $b$ and $r$ represent the blue and red particles, respectively. In this study, we do not consider compressibility of the fluid, and the conservation of the energy is not included.

For three-dimensional case, 3D19V model as shown in Fig.1 is employed, in which the arrows refer to the direction of the particle motion, and particles translate in 18 directions and including the rest particles. The local equilibrium distribution function is given by

$$f_a^{(0)} = F_p\rho\ [\ 1 - 2Bc_{a\alpha}u_\alpha + 2B^2 c_{a\alpha}c_{a\beta}u_\alpha u_\beta + Bu^2 - \frac{4}{3}B^3 c_{a\alpha}c_{a\beta}c_{a\gamma}u_\alpha u_\beta u_\gamma - 2B^2 c_{a\alpha}u_\alpha u^2] \quad (5)$$

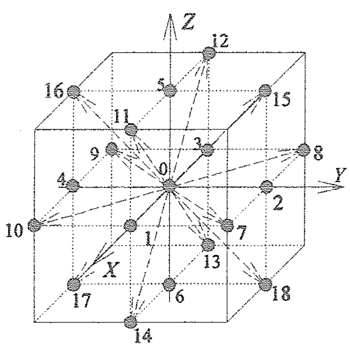



Fig. 1 Three-dimensional 19 velocity model

where

$$F_0 = \frac{1}{3}, \quad F_1 = \frac{1}{18}, \quad F_2 = \frac{1}{36}, \quad B = -\frac{3}{2c^2} \tag{6a,b,c,d}$$

and $F_0, F_1, F_2$ are factors corresponding to the rest, 1 to 6, and 7 to 18 particles, respectively., and $c = 1$ (the speed of particle 1 to 6). The local equilibrium distribution functions for blue and red particles have the same form. But the densities are replaced by $\rho_b$ and $\rho_r$, and the fluid velocities are defined below due to the body force.

## 3. Basic equations

### 3.1 Macroscopic equations

As shown in our previous study [9], the total density of the fluid

$$\rho = \rho_r + \rho_b \tag{7}$$

does not varies so much throughout the flow field. But the density variation is essential to the fluid motion, therefore we define a new density as

$$\rho' = \frac{\rho_b}{\rho_r + \rho_b} = \frac{\rho_b}{\rho} \tag{8}$$

which represents the ratio of the density of blue particles to the total density.

In order to introduce the gravitational force, however, the fluid velocity $\mathbf{u}$ in the local equilibrium distribution function for blue particles should be changed to $\mathbf{u} - \mathbf{g}\phi$, then

$$f_{bi}^{(0)} = f_{bi}^{(0)}(t, \rho, \mathbf{u} - \mathbf{g}\phi) \tag{9}$$

As to the Coriolis force, the local equilibrium distribution functions of both particles are changed as

$$f_i^{(0)} = f_i^{(0)}(t, \rho, \mathbf{u} - 2\mathbf{\Omega} \times \mathbf{u}\phi\tau) \tag{10}$$

where $\mathbf{\Omega}$ is the angular velocity of rotation of the system and $\tau$ is the time increment and is unity.

Ordinary Chapman-Enscog technique gives the continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \tag{11}$$

and the equation of motion employed the Boussinesq approximation as

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{1}{\rho_0} \nabla P + \rho' \mathbf{g} + (-2\mathbf{\Omega} \times \mathbf{u}) + \nu \nabla^2 \mathbf{u} \tag{12}$$

The pressure and the kinematic viscosity are expressed, respectively, as

$$P = \frac{1}{4}\rho, \quad \nu = \frac{1}{5}\left(\phi - \frac{1}{2}\right) \tag{13a,b}$$

If we define the temperature by

$$T = \frac{\rho_r}{\rho} \tag{14}$$

the relationship between the temperature and the density $\rho'$ is expressed as

$$T = 1 - \rho' \tag{15}$$

and the energy equation may be written as

$$\frac{\partial T}{\partial t} + (\mathbf{u} \cdot \nabla)T = k\nabla^2 T + g\frac{\partial}{\partial z}(1 - T) \tag{16}$$

where $k$ is the thermal conductivity and

$$\kappa = \frac{1}{5}\left(\phi - \frac{1}{2}\right) \tag{17}$$

In the above equation, the last term on LHS refers the effect that the temperature in upper space increase in time by the gravity because the gravity acts on only the blue particles and the population of the blue particles decreases. This effect is artificial and the effect can be negligible when we consider fast motions presented in this paper.

3.2 Non-dimensional parameters

The reference length is taken to the height of the flow field $L_z$ and the reference velocity is the following buoyancy velocity

$$U_B = \sqrt{g\frac{\Delta\rho'}{\rho'_0}L_z} \tag{18}$$

where $\Delta\rho'$ is the difference between the maximum and the minimum densities, $\rho'_0$ represents the maximum density. The parameters governing the gravity current are the Grashof number, the Ekman number, and the Rossby number, and defined respectively by

$$Gr = \frac{U_B^2 L_z^2}{\nu^2}, \quad E = \frac{\nu}{\Omega L_z^2}, \quad Ro = \frac{U_B}{\Omega L_z} \tag{17a,b,c}$$

Among these parameters, the relation $Ro = \sqrt{Gr} \times E$ holds, so that when we define the Grashof number, one of the two parameters, the Ekman and the Rossby numbers, is determined if the other is defined. In this paper, the Grashof number and the Ekman number are used.

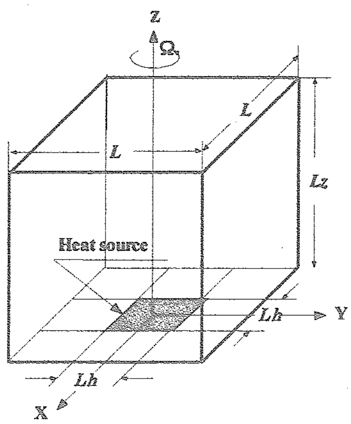



Fig.2 Natural convection in a cubic space in rotating system.

## 4. Numerical simulations

In this paper, we show that the present model can simulate stratified rotating flows by presenting the results of natural convection in a rotating system, which is a simple model of heat island, and rock-exchange gravity current which corresponds fundamental flows in sea breeze or thunderstorm out flow.

In this study, cubic lattice is used and the Cartesian co-ordinates are taken as follows. The $x$-$y$ plane is set horizontal, the $z$ axis is in vertical direction, and $x$ axis is in the flow direction for the gravity current. The axis of rotation is taken in the vertical direction and the rotation is anti-clockwise viewing from above.

### 4.1 Natural convection in a cubic space

A schematic situation of the flow is presented in Fig.2 and the heat source is set in the center of the bottom and the temperature is unity. On the other walls, no-slip and the temperature is 0.5. First the dependence of the number of the lattice is checked. Figure 3 shows the temperature distribution and the vertical component of flow velocity at almost steady state when the initial condition is 0.5 for temperature and 0 for flow velocity. The number of the lattice larger than $48\times48\times48$, the result does not change. The calculation hereby the $48\times48\times48$ lattice is used.

The typical flow pattern for $Gr=6.75\times10^5$, and $E=1.48\times10^{-3}$ is shown in Figs.4 and 5. In Fig.4 the velocity distribution in middle horizontal cross-section, and counter-clockwise circulation appears accompanying rising flow (the figure is not shown but easily understood) above the heat source which corresponds to the low-pressure region. Outside of this region there appears clockwise circulation which corresponds to the high-pressure region. The flow is in geostrophic state as shown in Fig. 5, in which the pressure gradient coincides with the fluid velocity, actually balances with the Coriolis force.

Some calculations are performed by fixing the Grashof and the Ekman numbers and changing gravitational force and other parameters. The results are

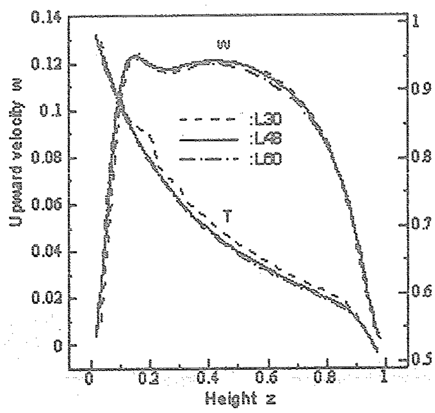



Fig.3 Dependence on the number of lattice

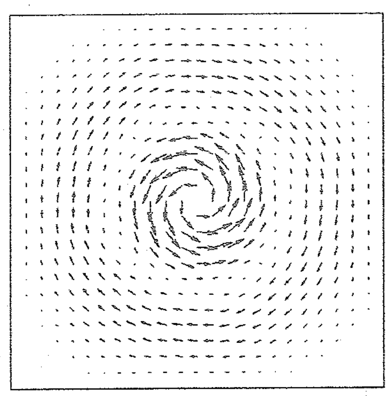


Fig.4 Horizontal velocity distribution

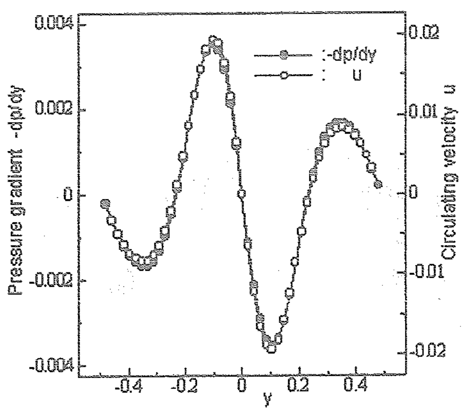



Fig.5 The relationship between the pressure gradient and the peripheral velocity in Geophysical state

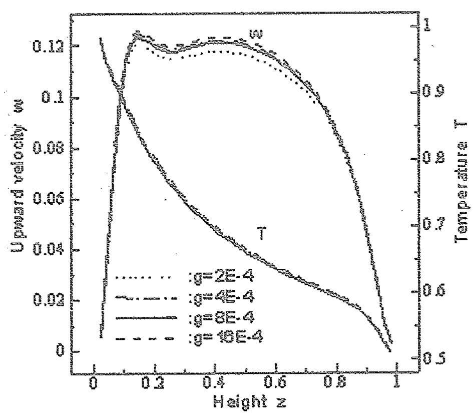



Fig.6 Similarity of flow pattern changing the gravitational force but with the same *Gr* and *E*.

shown in Fig 6, and the flow patterns are almost the same even changing the gravity force by 8 times, and therefore this model, in which the gravitational force acts on one kind of particles and the Coriolis force on both particles, is shown to be suitable for simulation of stratified rotating flows.

### 4.2 Gravity current

Gravity currents are horizontal convection due to density difference of two fluids in the gravity field. The heavy fluids runs below right fluids horizontally and the right fluid runs above the heavy fluid in reverse direction. The gravity currents often occur in many flows of nature for instance the thunderstorms, the sea breezes and so on. Benjamin [10] firstly analyzed the flow for inviscid fluids and determined the shape of the front. The shape is different from Benjamin's one when the non-sjip condition is employed on the ground. Many reports have been presented and are summarized in Simpson's report [11] and his book [12]. Recently, Haetel et.al.[13] presented direct numerical simulations in three dimensional case, but without rotation, and discussed the shape of front in detail.

We present the results for the rock-exchange flows, in which the heavy and light fluids are initially separated by a vertical gate as shown in Fig.7, and horizontal flows are generated by removing the gate.

A flow pattern without rotation for $Gr = 5.\times 10^7$ at 1600 time step is shown in Fig.8, which shows equi-density surface $\rho' = 0.5$, and the lobes and clefts are clearly seen in the front of the current head. In this calculation, the number of the lattice is $360\times 80\times 30$ and boundary conditions on the top and bottom are non-slip and that on the side walls are slip.

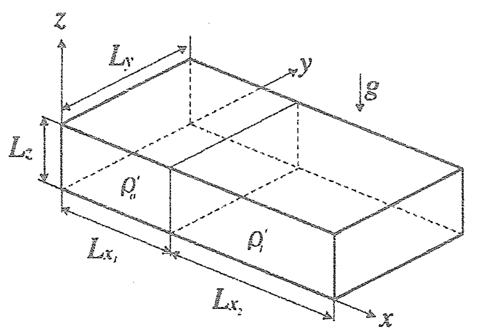



Fig.7 Initial condition of rock exchange flow

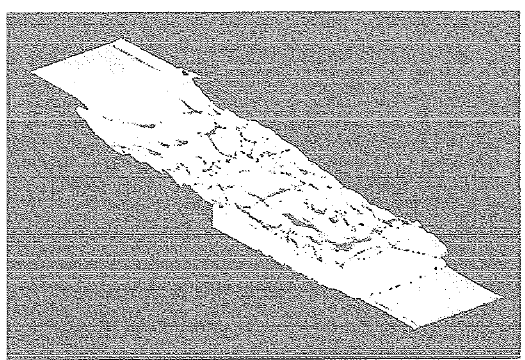
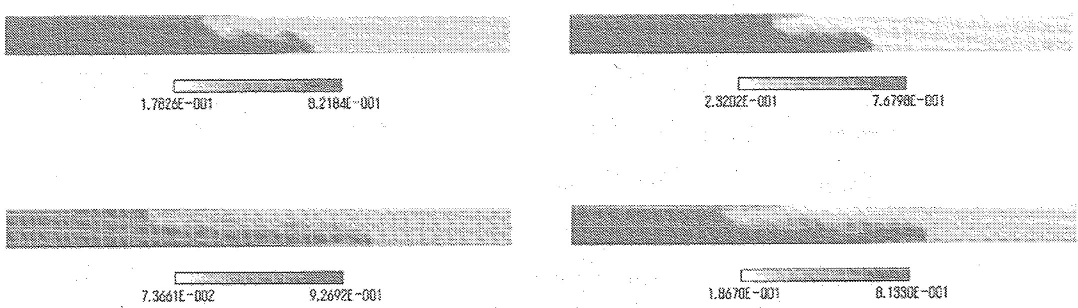


Fig.8 Equi-density surface of gravity current

Flow patterns in the vertical cross-section for two different Grashof numbers $Gr = 5.\times 10^7$ and $Gr = 2.\times 10^5$ are shown in Fig. 9, and it is seen that the Kelvin-Helmholtz type unstable waves on the top of the heavy fluid become stronger as the Grashof number increases. It is noted that the patterns of the right fluid and the heavy fluid are almost symmetric with respect to the central point of the flow region, because the flow is governed by the equations of the Boussinesq approximation in (12).




(a) Flow pattern for $Gr = 5.\times 10^7$ (b) Flow pattern for $Gr = 2.\times 10^5$

Fig.9 Density distributions for different Grashof numbers at 400 and 800 time steps

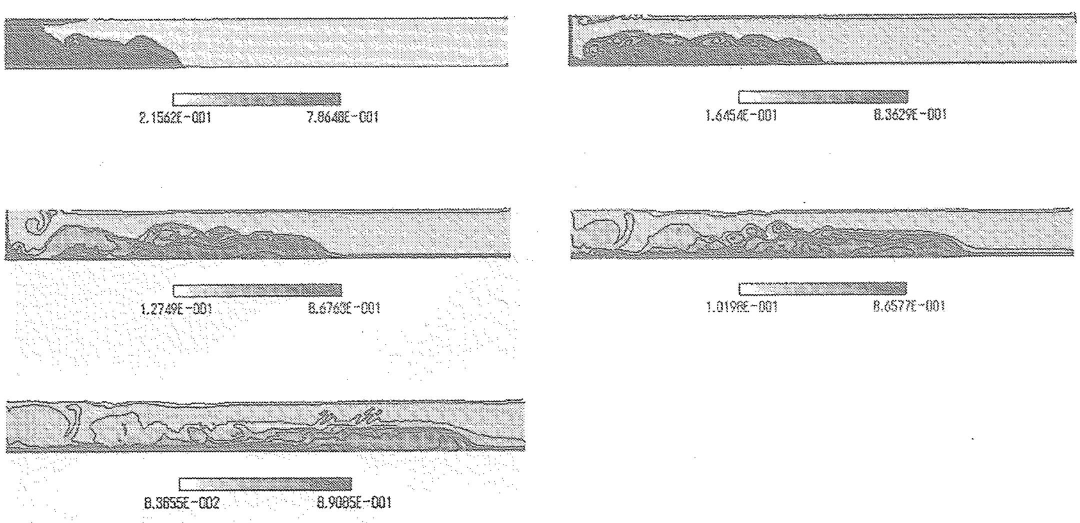



Fig.10 Density distributions at 600, 1200, 1800, 2400, 3000 time steps for initially different

When the initial region of the heavy fluid is much smaller than that of the light fluid ($L_{x1}/L_{x2}=1/4$, and the number of lattice is $360\times80\times40$), the current length behind the head of the heavy fluid decreases as time [11] as shown in Fig.10 for $Gr=5.\times10^7$. Figure 11 shows the three-dimensional flow pattern at the head for the same flow, the lobes and the clefts are more clearly seen.

When the system is rotating the flow pattern is completely different from non-rotating cases. The flow patterns in the cross-section for the flow of $Gr=5.\times10^7$, $E=8.0\times10^{-4}$ are shown in Fig. 12. In this calculation, the boundary condition on the side walls are periodic.

This figure shows that the current stops its running at some distance, which corresponds to Rossby's radius, and in the horizontal plane of middle height the

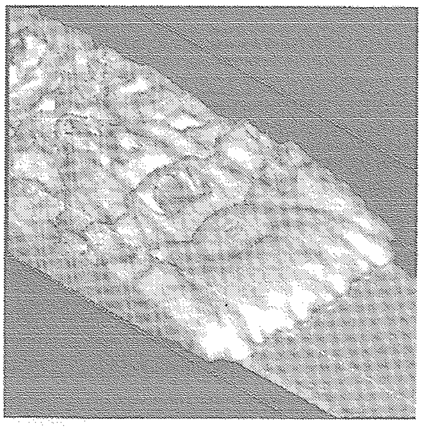


Fig.11 Three-dimensional flow pattern at the current head of the same flow in Fig.10.

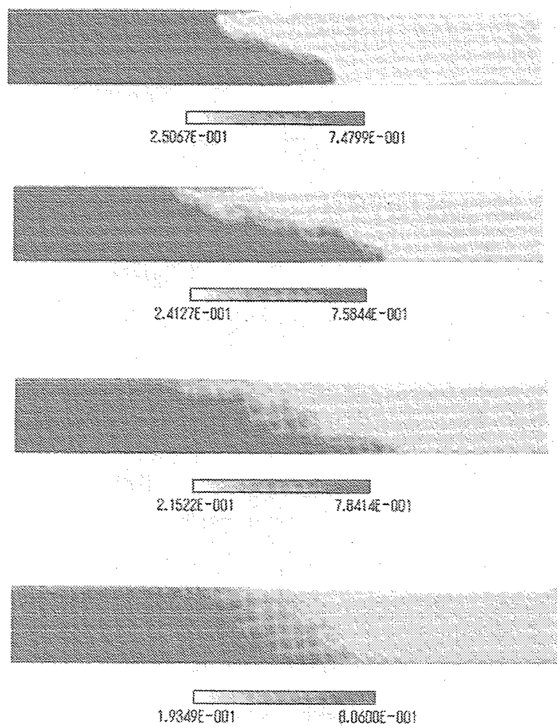



Fig.12 Density distributions for rotating case at 400, 800, 1200, 1600 time steps

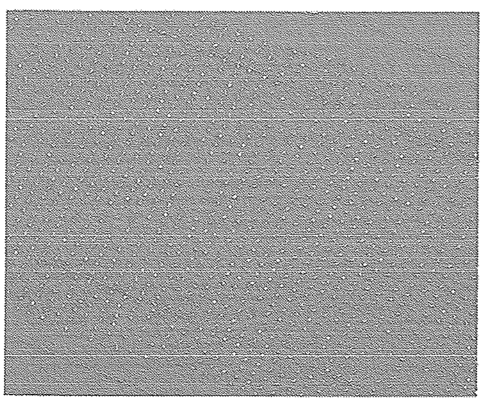


Fig.13 Horizontal velocity distribution near the current front in the middle height cross-section at 800 time step

velocity vectors are almost parallel to the current front as shown in Fig.13. This pattern can be considered to establish the geostorophic flow in which the pressure gradient and the Coriolis force are balanced.

## 5. Conclusion

Two kinds of particle model of the lattice Boltzmann method in which the gravitational force acts on one kind of particles and the Coriolis force on both particles, is applied to stratified flows in rotating system, and natural convections and gravity currents are successively simulated.

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