THE PATTERN OF THREE-DIMENSIONAL THERMOCAPILLARY CONVECTION

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ABSTRACT

The half-zone (HZ) model is simplified from the floating zone crystal growth process, and is widely used to study thermocapillary convection. In the present work, a new evolutional HZ model is proposed. Three-dimensional unsteady thermocapillary convection at high-Pr in this model is investigated. A comparison of the flow pattern after the first instability in between the present and the HZ models is conducted.

INTRODUCTION

The unbalanced surface tension caused by temperature and concentration gradients arises in many materials processes and leads to a significant thermocapillary convection. Since the importance of thermocapillary convection in floating zone crystal growth was pointed out in 1970s, thermocapillary convection in the HZ model has been widely studied in the recent two decades.

Thermocapillary convection is steady and axisymmetric for Marangoni number Ma less than the critical value Ma_c in the HZ model. The first instability is announced by the appearance of azimuthal flow with an increment of Ma above Ma_c, and the axisymmetry is broken. Based on the spatio-temporal features, the instability mechanism is believed to be different between high- and low-Pr (Pr: Prandtl number) thermocapillary convection.

Based on experimental results of thermocapillary convection in high-Pr half-zone liquid bridge, an important correlation between azimuthal wave number m and aspect ratio As, $m*As \approx 2.2$, was reported [1]. In Ref.[2], explanations on the instability mechanism for high-Pr flow, on observed structure characteristics and especially on the nature of correlation between As and m were presented.

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To better understand thermocapillary convection, three-dimensional unsteady thermocapillary convection in the proposed new model is conducted, and then it is compared with thermocapillary convection in the HZ model.

PHYSICAL AND MATHEMATICAL MODELS

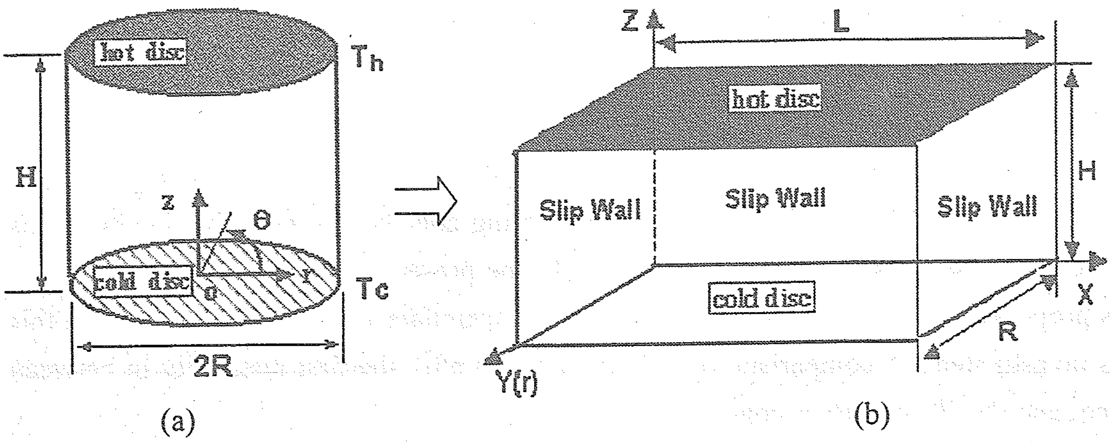

Figure 1. (a) The Half-Zone (HZ) model and (b) Box-Like (BL) model

The HZ model is a cylindrical liquid bridge held by surface tension and suspended between two discs as shown in Figure 1(a). If one cuts a half-zone through a θ plane, then stretches the axis to a plane, a box-like (BL) model as in Figure 1(b) is formed. Comparing to the HZ model, the curvature of the free surface becomes zero in the BL model, and cycle condition along the azimuthal direction in the HZ model is replaced by two slip wall conditions. Using the scales H, H^2/κ, κ/H, $\rho\kappa^2/H^2$ for length, time, velocity, and pressure, where ν, κ and ρ stand for kinematic viscosity, thermal diffusivity and density, the governing equations take the form:

$$\nabla\cdot\mathbf{u}^* = 0, \quad (1)$$

$$\partial\mathbf{u}^*/\partial t^* + \nabla\cdot(\mathbf{u}^*\mathbf{u}^*)-Pr\nabla^2\mathbf{u}^*=-\nabla P^* \quad (2)$$

$$\partial T^*/\partial t^*+ \nabla\cdot(T^*\mathbf{u}^*)-\nabla^2 T^*=0. \quad (3)$$

Here, $\mathbf{u}^*=(u^*, v^*, w^*)^T$, P^* and T^* denote the dimensionless velocity, pressure, and temperature. The dimensionless temperature is $T^*=(T-T_m)/\Delta T$ with $T_m=(T_h+T_c)/2$ and $\Delta T=T_h-T_c$. The important dimensionless parameters are the Prandtl number $Pr=\nu/\kappa$, aspect ratio $As1=R/H$, $As2=L/H$ and Marangoni number $Ma=\sigma_k\Delta TH/(\rho\nu\kappa)$.

For the BL model in Cartesian coordinates (x,y,z) as in figure 1(b), the applied boundary conditions are

$$\mathbf{u}^*=0,\ \ T^*=0.5 \text{ at } z^*=1 \quad (4)$$

$$\mathbf{u}^*=0,\ \ T^*=-0.5 \text{ at } z^*=0 \quad (5)$$

$$v^*=\partial w^*/\partial y^*+Ma\partial T^*/\partial z^*=\partial u^*/\partial y^*+Ma\partial T^*/\partial x^*=\partial T^*/\partial y^*=0 \ \text{ at } y^*=As1 \quad (6)$$

$$u^*=\partial T^*/\partial x^*=0 \quad \text{at } x^*=0 \text{ and As2} \tag{7}$$

$$v^*=\partial T^*/\partial y^*=0 \quad \text{at } y^*=0 \tag{8}$$

The dynamical system is discretized using the finite volume method with staggered grids and solved by an improved SIMPLE algorithm.

RESULTS AND DISCUSSIONS

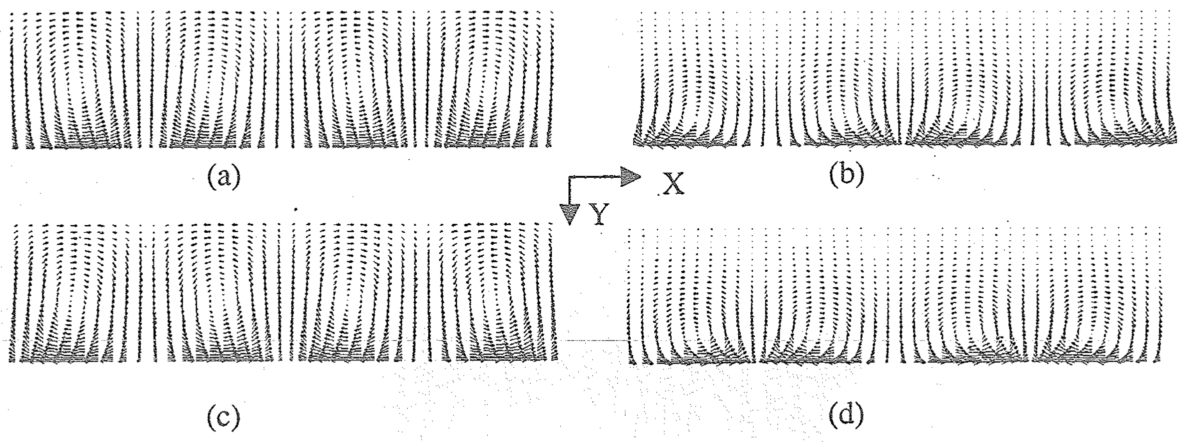

Figure 2 Disturbance flow in z=H/2 plane at (a) $t=\tau_0$, (b) $t=\tau_0+1/4\tau_p$, (c) $t=\tau_0+1/2\tau_p$, and (d)$t=\tau_0+3/4\tau_p$ for As1=1, As2=4, Pr=7 and Ma=10000

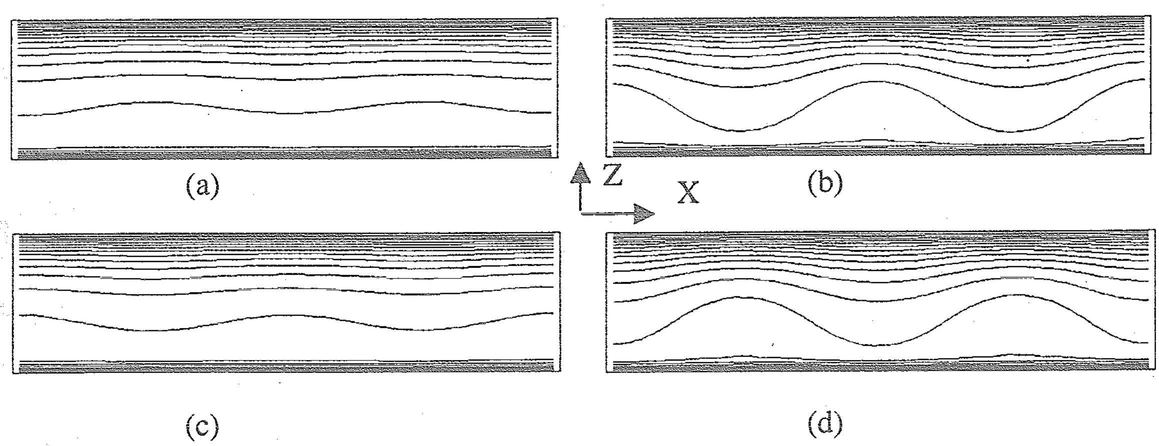

Figure 3 Temperature contour plot ($\Delta T^*=0.025$) on the free surface at (a) $t=\tau_0$, (b) $t=\tau_0+1/4\tau_p$, (c) $t=\tau_0+1/2\tau_p$, and (d)$t=\tau_0+3/4\tau_p$ for As1=1, As2=4, Pr=7 and Ma=10000

As in the HZ model, the unbalanced surface tension drives surface flow from hot to cold discs in BL model, and this surface flow penetrates into bulk of liquid in some degree due to viscosity. The back-flow is brought about by continuity. This is the basic structure for thermocapillary convection with $Ma<Ma_c$, and the basic vortex flow locates in the yz-plane. With an increment of Ma ($Ma>Ma_c$), the steady thermocapillary convection looses stability to become 3D oscillatory convection. For the convenience of describing the flow structure, the disturbance quantity is defined as a local deviation from the x-direction averaged value: $q' = q - \int_0^{As2} q dx / As2$. (q represent a dimensionless physical quantity, such as u^*, v^*, w^*, T^*). Figure 2 is the disturbance flow in z=H/2 cut for As1=1, and As2=4 and figure 3 is the temperature contour plot on the free surface. It exhibits 4-disturbance vortices structure in the z plane. The disturbance vortex grows and decays periodically, and the cold and hot temperature spot alone x direction are alternated on every half of period, and also the flow direction of disturbance vortex in z-cut. The disturbance surface flow directs from hot to cold spots in x-direction. The directions of adjacent vortices are contrary. By checking the temperature along line (y=R ∩ z=H/2) on the free surface, a standing wave-like oscillation is observed as shown in Figure 4. By applying Fast Fourier Transfer to the temperature-time curve, one basic frequency and its harmonics are obtained, and therefore a periodic oscillatory convection is confirmed.

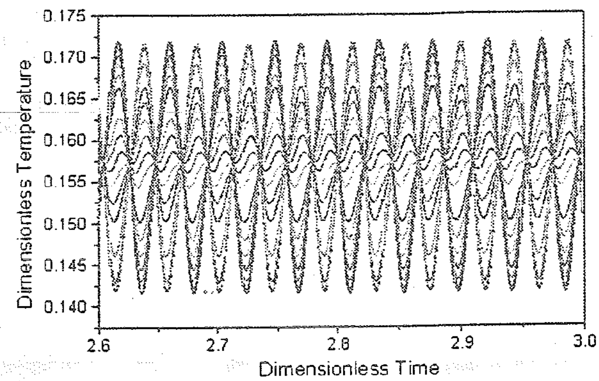

Figure 4 Temperature-time curve alone line (y=R ∩ z=H/2) on the free surface for Pr=7 and Ma=10000.

By checking the flow pattern for Ma=10000, As1=1 and As2=3, a 3-vortices structure as shown in Figure 5 is observed. The disturbance vortex grows and decays periodically, and a pulsating oscillatory convection is exhibited as shown in figures 5 and 6. For Ma=10000, As1=1 and As2=2, a pulsating oscillatory convection with 2 disturbance vortices in z-cut is observed.

In this study, As1 is fixed as 1, and therefore the characteristic size of the basic vortex in the yz-plane can be evaluated as H. The result demonstrates the number of disturbance vortices n depends on As2 for fixed As1, and the characteristic size of the disturbance vortex in a z=0.5 cut is about H as basic vortex in the yz-plane. The instability of the convection is accompanied with the appearance of disturbance vortices in a z-cut, which are perpendicular to the imposed temperature gradient. A model as figure 7 can describe the basic feature of the three-dimensional structure. The characteristic size of the disturbance vortex is about the characteristic size H of basic vortex as observed in the Figures 3 and 5. This evaluated disturbance vortex size (approximate equals size of basic vortex) is the important assumption for the derived correlation between As=H/R and the azimuthal wave number: As*m≈1.57~3.14 in the HZ model in Ref.[2].

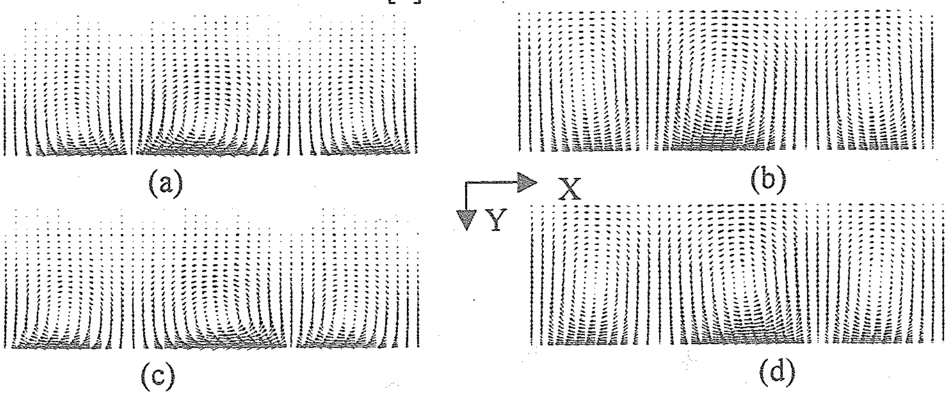

Figure 5 Disturbance flow in z=H/2 plane at (a) $t=\tau_0$, (b) $t=\tau_0+1/4\tau_p$, (c) $t=\tau_0+1/2\tau_p$, and (d)$t=\tau_0+3/4\tau_p$ for As1=1, As2=3, Pr=7 and Ma=10000

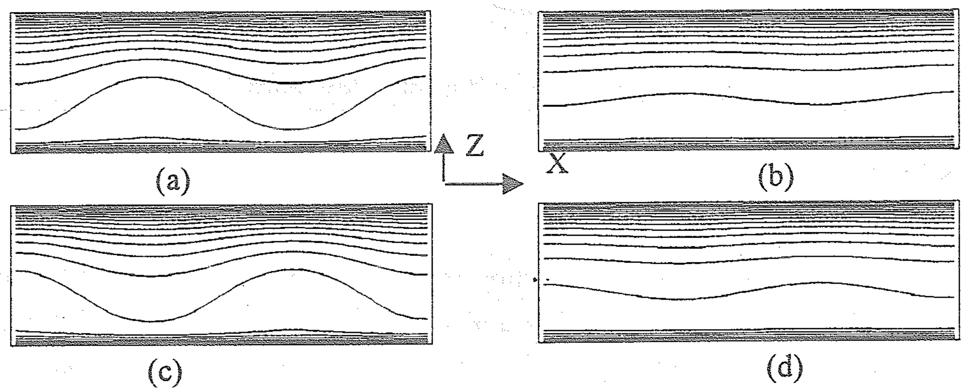

Figure 6 Temperature contour plot ($\Delta T^*=0.025$) on the free surface at (a) $t=\tau_0$, (b) $t=\tau_0+1/4\tau_p$, (c) $t=\tau_0+1/2\tau_p$, and (d)$t=\tau_0+3/4\tau_p$ for As1=1, As2=3, Pr=7 and Ma=10000

Comparing with the flow structure of pulsating oscillation in the HZ model as in Refs.[2,3], we can find that the basic flow structure features are similar, therefore the instability mechanism should be the same for high-Pr thermocapillary convection in both HZ and BL models. The physical explanation in Ref.[2] is also suitable to the observed structure characteristics here. This means that the free surface curvature and cycle condition along azimuthal direction in the HZ model is not important for the instability for high-Pr flow.

An important difference of three-dimensional thermocapillary convection between the HZ and BL models is that the disturbance vortex appears always in pairs in the HZ model, but the number of disturbance vortices in the BL model can be odd or even, which depends on As2 with fixed As1 as in Figures 2 and 5. The cycle condition along the azimuthal direction in the HZ model is replaced by two slip wall conditions in the x direction in the BL model, and this is believed to be reason causing the above difference.

In addition, for Ma=10000, As1=1 and As2=4, if Ma is increased to Ma=20000 and 30000, respectively, the convection is observed to be still dominated by pulsating oscillation in the time span under consideration. This result is different from the case in the HZ model, in which the pulsating oscillation tends to be replaced by a rotating oscillatory convection with a small further increment of Ma as in Refs.[3]. This difference is also believed to be caused by the disappearance of cycle condition along in the BL model.

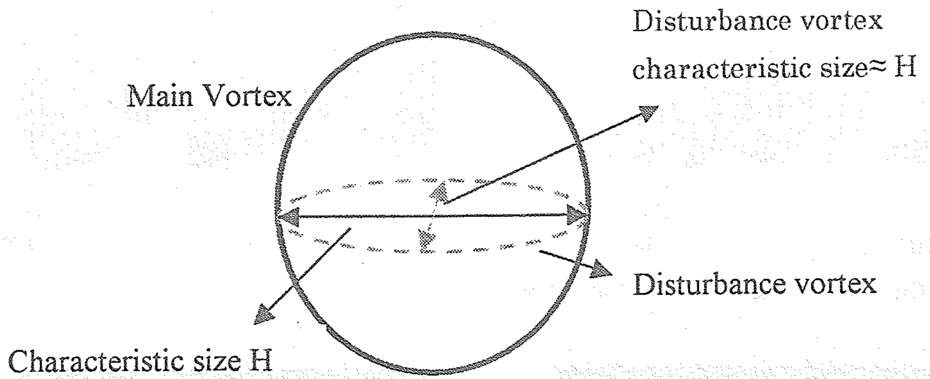

Figure 7 A model to describe flow pattern.

CONCLUSION

Three-dimensional thermocapillary convection is investigated in a deformed HZ model, the BL model. The disturbance vortices in the plane, which is perpendicular with the imposed temperature gradient, are observed after instability. The number of vortices depends on As2 for fixed As1. Pulsating oscillatory convection with standing wave like feature is observed, which is similar with the pulsating oscillation found in the HZ model. This result suggests the possibility to study high-Pr thermocapillary convection in a box model experiment instead of the HZ model to avoid the disadvantage of easy breaking of liquid bridge, although a further study on the effect of non-slip wall in real experiment rather than the slip wall condition in the BL model should been further investigated.

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