Efficient time- and frequency-domain simulation methods for vibro-acoustics and flow acoustics

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Overview
- Who we are
- Introduction:
  - Flow-induced acoustics and vibrations
  - Importance of virtual (acoustic) tools
- Mathematical formulations:
  - Flow-induced vibrations
  - Flow-induced acoustics
- Numerical solution strategies (@ KU Leuven):
  - for time-domain LEE
  - for frequency-domain vibro-acoustics
- Conclusions

Who we are

Our team
- KU Leuven (1425)
  - Department of Mechanical Engineering
  - Division of Production engineering, Machine design and Automation (PMA)
  - Noise and Vibration Research Group (MOD)
- Research staff
  - 5 academic and 1 associated
  - 1 industrial research manager
  - 11 postdoctoral researchers
  - 61 PhD incl. 10 industrial PhD res.
- Areas of research
  - vibro-acoustics
  - aero-acoustics
  - multi-body dynamics
  - smart system dynamics
  - structural reliability & uncertainty
- Application domains
  - energy and environment
  - transport and mobility
  - health
  - advanced manufacturing
- Conclusions

Importance of virtual (acoustic) tools

Modern product design:
- Ever expanding and often conflicting design criteria
- Shortening of the design cycle
- Acoustic performance: growing importance
  - Customer demands
  - Legal regulations

Numerical prediction techniques (virtual prototyping)
- (+) Faster and cheaper than real prototype testing
- (+) Sensitivity analyses
- (+) Acoustic evaluation at all stages of design

Conclusions
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Mathematical formulation
Flow-induced vibrations

• Aerodynamic excitation:
  ° Unsteady flow
  ° Stationary
  ° Homogeneous
  ° Fully developed
  ° Zero mean pressure gradient
• Structural vibrations:
  ° Out-of-plane bending
  ° Weak (one-way) coupling between aerodynamic field and elastic vibrations in the structure

Mathematical formulation
Flow-induced acoustics

• Aerodynamic noise generation: unsteady flow features
• Acoustic waves: time-harmonic compressibility waves

governed by the same type of equations: Navier-Stokes equations

Direct Noise Computation (DNC)

• Use scale-resolved CFD techniques to solve the Navier-Stokes equations directly

Scale discrepancies in CAA

<table>
<thead>
<tr>
<th>Aerodynamic field</th>
<th>Acoustic field</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smallest length scale</td>
<td>Turbulent structures: $l_t = D \ell_{Re}^{0.6}$ ($\mu$m)</td>
</tr>
<tr>
<td>Propagation speed</td>
<td>Convected with the mean flow velocity $U_{f}$</td>
</tr>
<tr>
<td>Propagation distance</td>
<td>Short (~cm)</td>
</tr>
<tr>
<td>Energy level</td>
<td>$p': 100$–$1000$ Pa</td>
</tr>
<tr>
<td>Velocity</td>
<td>$v': 1.0$ m/s</td>
</tr>
</tbody>
</table>

!! low Mach, high(er) Reynolds
Direct Noise Computation (DNC)

- Most restrictive requirements of aerodynamic and acoustic variables:
  - Fine grid (turbulent length scales)
  - Large grid (acoustic propagation distance)
  - Numerical scheme with low dissipation and dispersion error
- ... extremely demanding, not applicable for industrial applications

Hybrid methodologies

- Domain decomposition:
  - Source region: noise generation
  - Acoustic domain: noise propagation
- Coupling technique:
  - Equivalent sources (Acoustic analogies)
  - Boundary conditions (Kirschhoff, Flouca Williams-Hawking)

KU Leuven research activities

- Compressible Navier-Stokes equations:
  - All flow-acoustic interaction phenomena
  - Convective noise propagation
  - Non-linear acoustic propagation effects
  - Visco-thermal effects
- Open simplifications are possible:
  - LNSE, LEE, APE, CWE, AWE, ...

Acoustic propagation equations

Linearized Navier-Stokes Equations (LNSE)

Assumptions:
- Variable decomposition is possible and mean flow field is known (from e.g. RANS-simulation)
- Non-linear effects can be neglected: linearization of the equations
- The acoustic waves do not influence the mean flow field: one-way interaction
- Viscous effects are considered small enough to assume isentropic flow

Compressible Navier-Stokes Equations:

- Continuity:
  \[
  \frac{\partial \rho'}{\partial t} + \frac{\partial (\rho'u')}{\partial x} + \frac{\partial (\rho'v')}{\partial y} + \frac{\partial (\rho'w')}{\partial z} = 0
  \]
- Momentum:
  \[
  \frac{\partial (\rho'u')}{\partial t} + \frac{\partial (\rho'uu')}{\partial x} + \frac{\partial (\rho'uv')}{\partial y} + \frac{\partial (\rho'uw')}{\partial z} = 0
  \]
  \[
  \frac{\partial (\rho'v')}{\partial t} + \frac{\partial (\rho'vu')}{\partial x} + \frac{\partial (\rho'vv')}{\partial y} + \frac{\partial (\rho'vw')}{\partial z} = 0
  \]
- Energy: isentropic flow
  \[
  \frac{\partial (\rho'u)'}{\partial t} + \frac{\partial (\rho'u'v)}{\partial x} + \frac{\partial (\rho'u'v)}{\partial y} + \frac{\partial (\rho'u'w)}{\partial z} = 0
  \]
Acoustic propagation equations

Linearized Euler Equations (LEE):
additional assumption: viscous effects negligible

- Einstein notation:
  \[ \frac{\partial p}{\partial t} + \nabla \cdot (p V) = 0, \]
  \[ \frac{\partial V}{\partial t} + (V \cdot \nabla) V = -\frac{1}{\rho} \nabla p + \nu \nabla^2 V, \]
  \[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho V) = 0, \]

- Vector notation:
  \[ \frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{V}, \]

- Matrix notation:
  \[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0, \]
  \[ \frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{V}, \]

- Additional assumption: viscous effects negligible

Acoustic propagation equations

Convective Wave Equation (CWE):
additional assumption: uniform mean flow

- Linear acoustic wave equation:
  \[ \frac{\partial^2 p}{\partial t^2} - c^2 \nabla^2 p = 0, \]
  \[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0, \]
  \[ \frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{V}, \]

- Acoustic Wave Equation (AWE):
  \[ \frac{\partial^2 p}{\partial t^2} - c^2 \nabla^2 p = 0, \]
  \[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0, \]
  \[ \frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{V}, \]

Comparison of propagation equations

<table>
<thead>
<tr>
<th>NS</th>
<th>Linearized NS</th>
<th>Euler</th>
<th>LEE</th>
<th>CWE</th>
<th>AWE</th>
</tr>
</thead>
<tbody>
<tr>
<td>non-linear effects</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>viscous effects</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>type of mean flow</td>
<td>any</td>
<td>any</td>
<td>any</td>
<td>uniform</td>
<td>rest</td>
</tr>
<tr>
<td>convective: refraction</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>convective: amplification</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>convective: wavenumber shift</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>convective: directivity change</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
</tbody>
</table>

Simulation of acoustic propagation:

- equations defined in **time-domain**:
  - natural description of source mechanisms
  - convenient interfacing with source information from unsteady CFD
  - efficient formulation for broadband and transient phenomena

- assuming time-harmonic behaviour: **frequency domain**
  - faster
  - only steady-state behaviour

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Overview spatial discretization:

Modeling tools for the LEE

Finite differences

Basic principle:

\[ \frac{\partial U}{\partial t} + \nabla \cdot (C U) = S + \text{boundary conditions} \]

Add artificial selective damping to avoid 'unphysical' and 'unstable' high-frequency oscillations.

Finite differences + straightforward implementation
- difficult BCs on large stencils
- difficult to change integration scheme
- low to moderate geometrical complexities
- stability restrictions on time step

Discontinuous Galerkin

Galerkin approach:
- Multiplying LEE with base function \( b_k \)
- Integrating over each element \( \Omega \)
- Weighted residual formulation + partial integration

\[ \int_{\Omega} b_k \nabla U \cdot \nabla U - \int_{\Omega} b_k \nabla U \cdot \nabla \phi = 0, \quad k = 1, \ldots, N \]

Only communication between elements through inter-element flux

some key properties
- easy parallelization
- straightforward implementation of B.C.
- arbitrary order of accuracy
- grid flexibility
- efficiency
- storage cost for 5 DOF’s
- quadrature demands CPU-time

KU Leuven innovations
1. optimized Runge Kutta schemes
2. time domain impedance boundary condition based on recursive convolution
3. source mean flow mapping and filtering
4. Random Particle Mesh (RPM) method
5. modelling tools for the Linearized Navier-Stokes Equations (LNSE)
KU Leuven research activities

Hybrid methods: Coupling techniques
- Hybrid methods: domain decomposition
  - Fine CFD grid (source domain)
  - Coarser acoustic grid (propagation domain)
- Coupling: mapping of data from the CFD grid to the acoustic grid
  - Mean flow field
  - Source data

Interfacing schemes: Mean flow mapping

Tandem cylinders case

Mean flow from RANS
(250,414 cells)

Interfacing schemes: Mean flow mapping

Tandem cylinders case

- Different orders used for the fitting mesh and different cut-off degrees for the filter
- Error values associated with the intermediate mesh
- Error less than 0.1%

Source region modelling: Numerical
Noise generation: unsteady flow field
- Scale-resolving simulations (DNS, LES)
  - Computationally demanding
  - Conservative interfacing scheme (mapping) needed
- Steady flow simulations (RANS) + stochastic reconstruction
  - Reconstrunct unsteady variables from steady RANS

Stochastic Methods
- SNGR (frequency-domain)
- RPM (time-domain)
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Modelling tools for the Helmholtz equation

**Finite Element Method**

- geometrical flexibility
- frequency independent submatrices
- banded sparse matrices
- low-threshold automation
- (very) large matrices
- computationally demanding
- interpolation/dispersion error

**Boundary Element Method**

- unbounded domains
- small matrices
- only boundary mesh
- dense matrices
- frequency dependent
- computationally demanding
- interpolation/dispersion error

Acoustic sources:

- structured RPM mesh
- (Smaller) source zone

Modelling tools for the Helmholtz equation

Finite Element Method

\[ \nabla^2 p(r) + \frac{\omega^2}{c^2} p(r) = Q(r) \]

+ boundary conditions

\[ p(r) = \sum_{i} N_i(r) p_i \]

sufficient number of elements per wavelength...

... frequency limitation

Boundary integral formulation

\[ p(r) = \sum_{\Gamma} \left[ A \right] \left[ p_i \right] \]

+ boundary conditions

\[ p(r_0) = \sum_{\Gamma} N_i(r_0) p_i \]

\[ v_\text{in}(r_0) = \sum_{\Gamma} N_i(r_0) v_\text{in} \]
KU Leuven innovations

Modelling tools for vibro-acoustic simulation

1. efficient Wave Based Method (WBM) for Helmholtz problems

2. efficient inclusion of TBL excitation models

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KU Leuven innovations

Efficient Wave Based Method for acoustics

indirect Trefftz approach: approximation of the field variables by expansions of globally defined, exact solutions:

\[ p(x, y) = \sum_{j \in J} \Phi_j(x, y) + \tilde{p}_j(x, y) \]

acoustic wave functions:

\[ \Phi_j(x, y) = \frac{\cos(k_{xj}x)e^{-ik_{yj}y}}{k_{yj}^2} \]

particular solution (point source):

\[ \tilde{p}_j(x, y) = \frac{P_{j,0}(x)}{4} H_j^0(k_{yj}) \]

requirement: \( k_{xj}^2 + k_{yj}^2 = k^2 \) (1 solution!)

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Modeling tools for vibro-acoustics

Wave Based Method

**FEM and WBM:**

Same computational efforts (40 sec/frequency)

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KU Leuven innovations

Efficient Wave Based Method for thin plate bending

Kirchhoff equation:

Out-of-plane bending

Bending wave functions:

Particular solution (point force):

Requirement: \( k_{xj}^4 + k_{yj}^4 = k^4 \) (1 solution!)

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Response PSD from TBL PSD and wave admittance


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KU Leuven innovations

Efficient inclusion of TBL excitation

**Improvement 1:** Faster calculation using WBM

- Complex wave functions
- High efficiency
- Multiple RHS

**Improvement 2:** Generalised Corcos

- Tunable
- Low wavenumber decay
- Complex residue integration

→ Faster response prediction
Conclusions

- **Aerodynamic versus vibro-acoustics phenomena:**
  - Large disparity in energy/length/time scales
- **Numerical schemes:**
  - Balance between accuracy – stability - efficiency
- **Hybrid approaches:**
  - Noise/vibration generation – noise/vibration propagation
- **KU Leuven approaches:**
  - DG methods for LEE
  - Mapping techniques (for mean flow and sources)
  - RPM for stochastic source reconstruction
  - Wave Based Method for (TBL excited) vibro-acoustics

Thank you

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