NONLINEAR PHENOMENA OF COUPLED FLUTTER RESPONSES AND SELF-EXCITED FORCES OF A FLAT CLOSED-BOX BRIDGE DECK

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The nonlinear behaviors of post-flutter and corresponding self-excited forces of a flat closed-box bridge deck were investigated through spring-suspended sectional model (SSSM) tests. The sectional model was found to undergo nonlinear post-critical flutters, often called soft flutters, beyond the different linear flutter boundaries at common attack angles, i.e., 5°, ±3° and 0°, which exhibited self-limiting vibration behaviors in time domain and limit cycle oscillations (LCOs) in the final stable states of oscillation. The post-flutter LCOs occurred in a coupled mode which was dominated by the vibration in torsional degree of freedom (DOF) and coupled with obvious vibration in heaving DOF. The coupling effect of torsional-heaving vibrations increased with the rising wind speed in the post-critical region. The nonlinear self-excited lift force and torsional moment were measured by using four elaborate high-accuracy miniature three-component dynamic force balances installed inside the sectional model to reduce the inertia forces, and were validated by comparing the calculated and measured post-critical displacement responses. Both the measured self-excited lift force and torsional moment were found to contain significant higher-order multiple-frequency components, which indicate strong aeroelastic nonlinearity.

\textbf{Keyword}: flat closed-box deck, soft coupled flutter, self-limiting vibration, limit cycle oscillation, nonlinear self-excited forces

1. INTRODUCTION

Suppressing flutter instability is of great concern for long-span bridges. With continuous increase of span length and application of low-damping material, modern long-span bridges are becoming more susceptible to wind actions, which pose new challenges for engineers in guaranteeing aeroelastic stability. Up to now, widely-accepted approaches for flutter analyses of long-span bridges are commonly based on the linear unsteady self-excited force model proposed by Scanlan\textsuperscript{1)} in 1970s. Because the nonlinear aeroelastic effect is neglected in Scanlan’s linear model, it is only suitable for the cases of vibration with amplitudes small enough, in which the change of transient aerodynamic shape due to the vibration can be negligible. In this connection, the classical linear approaches for flutter analyses can only to be used to predict the linear boundary, i.e., the lowest critical point or wind speed of divergent-type flutter instability with rapidly-increasing amplitude to infinite. They are incompetent for predicting the post-critical responses of soft flutter, which is strongly nonlinear in both aeroelastic and structural behaviors and shows a vibration manner of limit cycle oscillation (LCO). However, for most long-span bridges with bluff decks the flutter responses or at least the post flutter responses are actually soft, namely, exhibit self-limiting vibration behaviors in time domain and LCOs in the final stable states of oscillation because of the nonlinearity of aerodynamic forces pertaining to the vibration responses. For instance, the flutter of Tacoma Narrows Bridge really happened in 1940 was not a strict divergent-type vibration, but was a LCO-type vibration with the largest torsional amplitude of about 35° and lasting for about 70 minutes before final collapse\textsuperscript{2}). Considering the LOC behavior

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of the soft flutter, flutter with small amplitude in short period should perhaps be allowed in the wind resistant
design of long-span flexible bridges, just like that already done for the vortex-induced vibration. It is thus very
necessary to investigate the nonlinear behavior of post flutter and to modeling mathematically the nonlinear
aeroelastic self-excited forces of typical bluff bridge decks under states of large-amplitude oscillation for the
purposes of reasonable prediction on post flutter responses of long span bridges.

The source of aeroelastic nonlinearity of flutter can be qualitatively attributed to the change of
transient relative attack angle of wind due to the bridge deck motion, which is equivalent to the change of
transient aerodynamic shape of the bridge deck relative to the direction of the transient equivalent resultant
wind. Therefore, the nonlinearity of self-excited force will become more and more significant with the
increasing vibration amplitude. The investigations of nonlinear aeroelastic effects and self-excited force
modeling have been carried out extensively in the field of aeronautics\textsuperscript{3}, wind turbine\textsuperscript{4} and energy harvesting\textsuperscript{5}.
In recent years, the related topics have attracted wide attentions in structural engineering. Noda\textsuperscript{6} found the
amplitude-dependence effect of linear flutter derivatives via forced vibration tests on two rectangular plates
with aspect ratio B/D=13 and B/D=150. Diana et al.\textsuperscript{7} studied the nonlinear hysteresis effect of a typical
closed-box section and later proposed a numerical approach to model the various aerodynamic nonlinearities
using a rheological mechanical model. Later, Liao et al.\textsuperscript{8} measured aerodynamic forces of thin airfoil and
closed-box bridge girder through large-amplitude forced vibration tests, and investigated the nonlinear
hysteresis effect and post-flutter behaviors based on the measured results. Amandolese\textsuperscript{9} studied the
post-flutter behavior of a flat plate and found that it would exhibit LCO characterized by significant
heaving-torsional coupling effect in post-critical state. Naprtek et al.\textsuperscript{10} studied the nonlinear post-critical
LCO of a rectangular 1:4 section and proposed a nonlinear self-excited forced model of Rayleigh or Van der
Pol with Duffing types. Wu and Kareem\textsuperscript{11} established a nonlinear convolution scheme using Volterra series to
model nonlinear self-excited force. Liu and Ge\textsuperscript{12} proposed a set of nonlinear differential equations with
internal states to model nonlinear and unsteady characteristics of bridge aerodynamics.

Considering that flat closed-box decks are often used on long-span bridges for its high performance of
flutter stability, such as the decks of Great Belt Bridge with a main span of 1624m, Xiangshan Harbor Bridge
with a main span of 688m, the nonlinear phenomena of the coupled flutter and the aeroelastic self-excited lift
force and torsional moment of a typical flat closed-box deck under post flutter states were investigated in this
study through SSSM tests in wind tunnel, and are to be discussed in this paper.

2. WIND TUNNEL TESTS OF POST-FLUTTER
(1) Brief information of wind tunnel and sectional model

The post-flutter tests of a flat closed-box deck were performed in TJ-2 wind tunnel. The wind tunnel is
a boundary layer tunnel of horizontal closed-circuit type with a testing section of 3m wide, 2.5m high and 15m
long. Wind speed can be continuously adjusted from 0.5 to 68.0 m/s. Fig.1 and Fig.2 display the sectional
model and its cross section. The general experimental setup is illustrated in Fig.3. The sectional model was
elastically supported by 8 helical springs through 2 arms. The horizontal long steel wires were used to
constrain the horizontal degree of freedom of the model. To avoid disturbing the two-dimensional flow around
the sectional model, the elastic supporting system (the helical springs, the suspending arms and the supporting
frame) and 3 displacement sensors were hided inside the two fairing walls parallel to the longitudinal axis of
the wind tunnel. The fairing walls were 0.12m thick, 2.4m high and 4.0m long. Its windward ends were of arc
shape to improve the flow quality between the two fairing walls.

The total length of sectional model is 1.920m. The total effective mass and mass moment of inertia are
23.221kg and 2.594kg·m\textsuperscript{2}, respectively, including the 1/3 mass of helical springs. The vertical and torsional
frequencies (f\textsubscript{0} and f\textsubscript{\alpha}) in still air were 1.245Hz and 2.075Hz, respectively. The structural damping ratios in
vertical and torsional modes (\zeta\textsubscript{h} and \zeta\textsubscript{\alpha}) in still air were nonlinear and vary, respectively, from 0.6% to 1.13%,
and from 0.08% to 0.37%, which will be further discussed infra.

All tests were carried out in smooth flow field and the Reynolds number Re varied within the range of
1.0×10\textsuperscript{5}~6.0×10\textsuperscript{7}. The maximal allowed ranges of the torsional and heaving vibration were ±6.6° and ±0.05m,
respectively by considering the limitation of the linear range of pre-tensioned springs and the linear measurement ranges of laser displacement sensors.

Figure 1: Spring-suspended sectional model between two fairing walls in TJ-2 wind tunnel

Figure 2: Cross section of sectional model

Figure 3: Schematic diagram of general experimental setup

(2) Test results of post-flutter behaviors

The post-flutter responses of the closed-box deck were investigated for four cases of wind attack angles of 5°, ±3° and 0°. For each case of attack angle, the sectional model vibration was random around its static equilibrium position when the wind speed was lower than a critical value \( U_{cr} \). Beyond this critical wind speed, the vibration became unstable and shows a divergent tendency with outward-concaved envelopes of oscillation crest and trough in the initial stage after a small excitation. But, with the increase of amplitude, the increase rate of the vibration amplitude became lower and lower until to zero when the vibration finally reached to a stable LCO state. The divergent tendency in the initial stage must be caused by the negative aerodynamic damping provided by the linear or even nonlinear components of self-excited forces whilst the self-limiting phenomenon must be resulted in by the positive aerodynamic damping provided by the high-order nonlinear components of self-excited forces. This kind of nonlinear flutter phenomenon is totally different from the divergent-type linear flutter, also called as “hard” flutter, as predicted by classical flutter.
theory, and is often called as 'soft flutter'.

Fig. 4 displays the root mean square (RMS) values of heaving and torsional displacements in different attack angles, where the upper arrows denote that the stable amplitudes were larger than the values of the relevant data symbols. One can then find from Fig. 4 that the post-flutter of the flat closed-box deck was featured by obvious heave-torsion coupling effect. The stable amplitude of heaving and torsional LCO both increased unrestrictedly with wind speed. The critical wind speed $U_{cr}$ was smaller and the LCO amplitude increased much slowly at a larger attack angle than at a smaller attack angle. These are mainly due to the fact that the larger the attack angle is, the bluffer the aerodynamic shape of the deck is, and the much significant the aeroelastic nonlinearity is.

![Figure 4: RMS values of stable responses of post-flutter](image)

Fig. 5 shows the post-critical LCOs in heaving and torsional degree of freedom at the attack angle of $5^\circ$ and the wind speed of $7.8\text{m/s}$. It can be found that both the heaving and torsional signals are not strictly harmonic and with slight higher-order components because of aerelastic nonlinearity, but dominated by the second mode corresponding to the torsional mode at zero wind speed, indicating that the observed post-critical instability occurred in the torsional mode. Fig. 6 further displays the evolution of vibration frequency in the heaving and torsional modes with wind speed. One can find that the vibration responses beyond critical wind speed $U_{cr}$ were only in torsional branch.

![Figure 5: Post-critical responses at $U=7.8\text{m/s}$ ($U*=U/ftB=5.466$, attack angle $5^\circ$)](image)

The coupling ratio of heaving and torsional responses $\gamma$ can be defined as $\text{RMS}(h)/\text{RMS}(\alpha)$, where $\text{RMS}()$ represents the RMS value of the relevant variable in parentheses; $b=B/2$ is the half width of bridge deck. Fig. 7 shows the variations of $\gamma$ with wind speed in the post-critical state for the attack angle of $5^\circ$ and $3^\circ$. It can be found that the coupling ratio increases with the wind speed in the post-critical region and the
variation laws are approximately same for different attack angles.

Figure 6: Evolution of vibration frequency with wind speed (attack angle 5°)

Figure 7: The coupling ratio of heaving and torsional degree of freedom in post-critical state

3. MEASUREMENT OF NONLINEAR SELF-EXCITED FORCES
(1) Improved force measurement technique

An improved technique, including the development of four elaborate high-accuracy miniature three-component dynamic force balances (see Fig.8) and the installation of the force balance inside the sectional model (see Fig.3 and Fig.9) to reduce the inertia forces, was adopted in this study to measure the nonlinear self-excited forces on a vibrating SSSM during the post-critical LCOs. The force balance was piezoelectric type with high sensitivity and small size of 0.035×0.05×0.05m. The weight of each balance is about 0.128kg. The linear range of vertical shear force is 12N and torque 0.9N·m with measurement error less than 4.57%F.S.

To further improve the measurement accuracy of self-excited force, the exterior ‘coat’ of the sectional model was separated into two 0.420m-long side segments and one 1.08m-long middle measurement segment. There were 1~2mm gaps between the middle and side “coat” to avoid any interference. The middle ‘coat’ was mounted on the internal rigid frame of the model through the 4 force balances while the side ‘coats’ were directly fixed on the rigid frame. Hence, only the dynamic forces on the middle ‘coat’ were measured. The ‘coats’ were made of light wooden plates stiffened by thin-walled duralumin to reduce the mass and thus the inertial force acting on the force balances as possible. The mass and mass moment of inertia of the middle ‘coat’ were 3.228kg/m and 0.0991kg·m²/m, respectively, which were only about 1/3.75 and 1/13.6 of the total ones of the SSSM system (12.094 kg/m and 1.351kg·m²/m), respectively.

The loading state of the middle exterior ‘coat’ during post-critical LCO was shown in Fig.10. From the dynamic equilibrium conditions of exterior ‘coat’, the self-excited lift and moment per unit length can be expressed as follows:

\[
M_{sc}(t) = M_{ms}(t) - M_{sc}^0(t) - L_f(t) \tag{1}
\]

\[
L_{sc}(t) = L_{ms}(t) - L_{sc}^0(t) - L_f(t) \tag{2}
\]
where $M_s(t) = -J_s \cdot \dddot{\alpha}(t)$ and $L_s(t) = -m_s \cdot \ddot{h}(t)$ are inertial moment and force acting on the exterior ‘coat’ per unit length. $M_{ms}(t)$ and $L_{ms}(t)$ are the total dynamic moment and force measured by the 4 balances, which can be expressed as:

$$
M_{ms}(t) = \left[ (M_{m1} + M_{m2} - M_{m3} - M_{m4}) + (F_{my1} - F_{my2} + F_{my3} - F_{my4}) \times b_m / 2 \right] / l_m
$$

$$
L_{ms}(t) = - (F_{my1} + F_{my2} + F_{my3} + F_{my4}) \times \cos \alpha_q + \left( -F_{mx1} - F_{mx2} + F_{mx3} + F_{mx4} \right) \times \sin \alpha_q
$$

Where, $M_{ms}, F_{myi}, F_{mx}(i = 1,2,3,4)$ represent the force signals measured by each force balance. $l_m$ is the length of middle ‘coat’, $b_m$ is the transverse distance between force balance, as shown in Fig.10. $\alpha_q$ represents the static angle of attack.

$$
M^0_{se}(t) = \left( \rho_a \right) \cdot \dot{\alpha}(t) - J^0 \cdot \dddot{\alpha}(t)
$$

where $J^0$ is the non-wind-induced additional moment of inertia, $c_{\alpha 0}$ is the non-wind-induced additional damping coefficient which is a nonlinear function of instantaneous torsional amplitude $\rho_a$.

Similarly, $L^0_{se}(t)$ in Eq.(2) represents the non-wind-induced aerodynamic force and can be expressed as:

$$
L^0_{se}(t) = - m_0 \cdot \ddot{h}(t) - c_{\dot{h}0} \left( \rho_h \right) \cdot \dot{\dot{h}}(t)
$$

where $m_0$ is the non-wind-induced additional mass, $c_{\dot{h}0}$ is the non-wind-induced additional damping coefficient which is a nonlinear function of instantaneous heaving amplitude $\rho_h$.

(2) Structural and non-wind-induced aerodynamic nonlinearities

It is known from Eq.(1)–Eq.(2) and Eq.(5)–Eq.(6) that non-wind-induced aerodynamic parameters, i.e., $J_s, m_s, c_{\alpha 0} \left( \rho_a \right)$ and $c_{\dot{h}0} \left( \rho_h \right)$, are necessary in extracting self-excited force $M_s(t)$ and $L_s(t)$. The nonlinear structural parameters, i.e., mechanical damping ratio $\xi_s (\rho_a)$, $\xi_h (\rho_h)$ and mechanical frequency $f_s (\rho_a), f_h (\rho_h)$, are also needed later when verifying the extracted self-excited force $M_s(t)$ and $L_s(t)$. The above non-wind-induced aerodynamic parameters can be identified from the measured force and displacement signals during free decay process in still air. The nonlinear structural parameters can be identified from the
free-decay displacement in still air\textsuperscript{15).}

The identified additional mass parameters, i.e., \( m_b \) and \( J_0 \), are 3.425 kg/m and 0.0300 kg·m\(^2\)/m, respectively, when the attack angle is 5\(^\circ\). Fig.11 illustrates the identified nonlinear structural and non-wind-induced additional damping ratios for both the heaving and torsional modes at different vibration amplitudes, where the additional damping ratios of heaving and torsional modes are represented by \( \xi_a(\rho_a) = c_{a0}(\rho_a)/\left[2\omega_{a0}\left(I + J_0\right)\right] \) and \( \xi_\alpha(\rho_\alpha) = c_{\alpha0}(\rho_\alpha)/\left[2\omega_{\alpha0}\left(m + m_b\right)\right] \), respectively. Fig.12 shows variation patterns of the heaving and torsional model frequencies with vibration amplitudes. As can be seen, the mechanical damping ratios, \( \xi_a(\rho_a) \) and \( \xi_\alpha(\rho_\alpha) \), and the non-wind-induced additional damping ratios, \( \xi_{a0}(\rho_a) \) and \( \xi_{\alpha0}(\rho_\alpha) \), increases all with the instantaneous amplitude, \( \rho_a \) or \( \rho_\alpha \), roughly in a linear manner. The mechanical frequency of the torsional mode \( f_\alpha(\rho_\alpha) \) decreases with the instantaneous torsional amplitude \( \rho_\alpha \) also in a linear way, whereas, that of the heaving mode \( f_a(\rho_a) \) decreases with the heaving amplitude \( \rho_a \) in a nonlinear way, indicating the softening effect of the SSSM system at large-amplitude states. Note that two different symbols where used in Fig. 11 and Fig.12, respectively, for the results obtained in the two repeated tests of free decay vibration, which agree well to each other, indicating the good repeatability of identified results.

\begin{align*}
\xi_a(\rho_a) &= c_{a0}(\rho_a)/\left[2\omega_{a0}\left(I + J_0\right)\right] \\
\xi_\alpha(\rho_\alpha) &= c_{\alpha0}(\rho_\alpha)/\left[2\omega_{\alpha0}\left(m + m_b\right)\right] \\
\xi_{a0}(\rho_a) &= c_{a0}(\rho_a)/\left[2\omega_{a0}\right] \\
\xi_{\alpha0}(\rho_\alpha) &= c_{\alpha0}(\rho_\alpha)/\left[2\omega_{\alpha0}\right]
\end{align*}

\( f_a(\rho_a) \) and \( f_\alpha(\rho_\alpha) \) are the identified modal frequencies of SSSM system in still air with vibration amplitude.

\begin{align*}
f_a(\rho_a) &= f_a(\rho_a) \\
f_\alpha(\rho_\alpha) &= f_\alpha(\rho_\alpha)
\end{align*}

It should be pointed out that the nonlinear parameters identified in still air cannot be directly used in the flowing air conditions, because the above parameters are also functions of vibration frequency, and according to Fig.6 it one be known that the vibration frequencies change with wind speed and the heaving
frequency can even switch between two modes. Therefore, the fitted nonlinear functions in Fig. 11 and Fig. 12 should be revised when used by considering the frequency change or switch. Taking $\xi_h(\rho_h)$ as an example, the fitted $\xi_h(\rho_h)$ in Fig. 11a corresponds to the heaving frequency $f_{h0}$, and when used in the case of heaving frequency $f_{hi}$, $\xi_h(\rho_h)$ should be revised to $\tilde{\xi}_h(\rho_h \cdot f_{hi} / f_{h0})$.

(3) Measured nonlinear self-excited lift force and torsional moment

After identifying the non-wind-induced aerodynamic parameters, the self-excited lift force $L_{se}(t)$ and torsional moment $M_{se}(t)$ can then be extracted from dynamic forces by subtracting the inertial forces and the non-wind-induced aerodynamic forces by Eq.(1)~Eq.(6). Fig.13 shows the time history of obtained self-excited lift force $L_{se}(t)$ and torsional moment $M_{se}(t)$ during post-critical LCO at $U=7.8$ m/s. As can be seen, the curve shapes of $L_{se}(t)$ and $M_{se}(t)$ are distorted from simple sinusoid. Fig.14 demonstrates the amplitude spectra of $L_{se}(t)$ and $M_{se}(t)$ at stable amplitude stage of post-critical LCO. Significant higher-order multiple-frequency peaks can be found. Both of the above phenomena indicate that the extracted $L_{se}(t)$ and $M_{se}(t)$ have strong aerodynamic nonlinearity.

![Figure 13: Time histories of Measured $L_{se}(t)$ and $M_{se}(t)$ at $U=7.8$ m/s ($U^*=U/ftB=5.466$, attack angle 5°)](a)

![Figure 14: Spectra of measured $L_{se}(t)$ and $M_{se}(t)$ at $U=7.8$ m/s ($U^*=U/ftB=5.466$, attack angle 5°)](b)

4. VALIDIFICATION OF MEASURED SELF-EXCITED FORCE

To validate the accuracy and reliability of the measured self-excited forces, the extracted time histories of $L_{se}(t)$ and $M_{se}(t)$ were directly applied to the governing equations of SSSM system, that is

$$\begin{align*}
(I + J_0)\ddot{\alpha} + \tilde{c}_a(\rho_a)\dot{\alpha} + \tilde{k}_a(\rho_a)\alpha &= M_{se}(t) \quad (7) \\
(m + m_0)\ddot{h} + \tilde{c}_h(\rho_h)\dot{h} + \tilde{k}_h(\rho_h)h &= L_{se}(t) \quad (8)
\end{align*}$$

Where, $\tilde{c}_a(\rho_a) = 2(I + J_0)\omega_{0a}\tilde{\xi}_a(\rho_a)$ and $\tilde{c}_h(\rho_h) = 2(m + m_0)\omega_{0h}\tilde{\xi}_h(\rho_h)$ are the nonlinear structural damping coefficients. $\tilde{k}_a(\rho_a) = (I + J_0)\tilde{\alpha}_a(\rho_a)$ and $\tilde{k}_h(\rho_h) = (m + m_0)\tilde{\alpha}_h(\rho_h)$ are the nonlinear structural stiffness coefficients. The superscript ‘~’ means the revision of the parameter values by considering frequency change of switch in the light of the corresponding fitted cured based on the tested data in still air. The instantaneous amplitude $\rho_a$ and $\rho_h$ are expressed as

$$\begin{align*}
\rho_a &= \sqrt{\alpha^2 + (\dot{\alpha} / \omega_{ai})^2} \\
\rho_h &= \sqrt{\dot{h}^2 + (h / \omega_{hi})^2}
\end{align*}$$

The post-critical responses of the SSSM system were then calculated numerically by Newmark-β
method according to Eq.(7) ~ Eq.(10), and were compared with the relevant measured results. Fig.15 shows the time history comparisons between the calculated and measured results of the heaving and torsional displacement responses at \( U=7.8\text{m/s} \) \( (U^*=5.466) \), while the RMS response comparisons between the calculated and tested results for different wind speeds in the tested post-critical region are plotted in Fig.16. Quite good agreements can then be found between the two sets of post-flutter responses with the discrepancies of RMS no more than 11%. Hence, the measurement accuracy and reliability of self-excited forces using the improved technique of interiorly-placed force balances are acceptable. The measured self-excited forces \( L_{se}(t) \) and \( M_{se}(t) \) can be further used to propose a feasible nonlinear self-excited force model, which suitable for large amplitudes of oscillation.

![Graph 15: Comparison between calculated and measured time histories of post-critical LCO at \( U=7.8\text{m/s} \) \( (U^*=U/\sqrt{f_0B}=5.466) \)](image1)

![Graph 16: Comparison between calculated and measured RMS of post-flutter responses](image2)

**5. CONCLUSIONS**

The nonlinear behaviors of the post-critical soft flutter and aerelastic self-excited forces of a flat
closed-box deck were investigated through sectional model tests. An improved testing technique of internally-placed force balances was developed to measure the self-excited forces during post-flutter and was verified to have a good accuracy. The test results showed that the flat closed-box deck would undergo coupled-flutter LCOs in post-critical region, which were in the second mode corresponding to the torsional mode in the zero wind case and featured by significant heaving-torsion coupling effect. Both the measured self-excited lift force and torsional moment during post-critical LCO contain strong nonlinear components. The mathematical models of the nonlinear self-excited lift force and torsional moment need to be further investigated in the next step.

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