Outline

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    - Cascaded LBM
  - Boundary condition
    - Wall boundary
    - Outer boundary
  - Building-Cube Method

- Numerical Results
  - Problem category 1-1 (2D)
  - Problem category 1-3 (3D)
  - Problem category 3-1 (3D)

- Conclusions
Background

■ CFD use:
- understanding the flow physics
- engineering design (especially in steady state)

■ Problems in current CFD
- Cost for unsteady flow simulation
  - High resolution/High order schemes
  - Restriction for $\Delta t$
  - Inner iteration of implicit time integration
  - Handling of massive output data
- Model dependency
  - DES, DDES, IDDES, Zonal DES, ...
  - RANS/LES switching parameter

- It is difficult to directly apply unsteady flow simulation for engineering design.

- Lattice Boltzmann Method has capability to overcome current CFD problem (?)

Algorithms of LBM

■ Governing equation: Boltzmann transport equation

$$\frac{\partial f_i}{\partial t} + \mathbf{e}_i \cdot \nabla f_i = \Omega_i \quad (i = 1, \ldots, b)$$

- $f$ : probability distribution function
- $\mathbf{e}$ : discrete set of velocities
- $\Omega$ : collision operator

■ Discretization on lattice:

$$f_i(r + \mathbf{e}_i dt, t + dt) = f_i(r, t) + dt \times \Omega_i(f_1, \ldots, f_b)$$

(i = 1, ..., b)

■ Lattice model used in this research: D2Q9, D3Q27

Ludwig Eduard Boltzmann (1844–1906)
Algorithms of LBM

- Relaxation parameter $\tau$ depends on the local grid size $\Delta x$.
  - $\frac{\Delta x_{\text{coarse}}}{\Delta x_{\text{fine}}} = n$ leads $\frac{\Delta t_{\text{coarse}}}{\Delta t_{\text{fine}}} = n$

- Temporal/spatial interpolation are necessary among difference size cells.

- Usually, non-equilibrium part of $f$ is rescaled.
  $$f_{\text{fine}} = f_{\text{eq,coarse}} + \left( f_{\text{coarse}} - f_{\text{eq,coarse}} \right) \frac{\Delta x_{\text{fine}} T_{\text{fine}}}{\Delta x_{\text{coarse}} T_{\text{coarse}}}$$

- Cascaded LBM is used for collision operator.
  - Satisfy Galilean invariance and has better accuracy/stability
  - Compute central moment defined by moving coordinate:
    $$\bar{M}_{p,q,r} = \sum_i \left( e_{ix} - u_x \right)^p \left( e_{iy} - u_y \right)^q \left( e_{iz} - u_z \right)^r \cdot f_i$$

  - Relation between Raw moment/Central moment
    $$\bar{M} = C^t \bar{M}$$

- 27 Central moments used in this research:
  - $\tau = 1$ is used for the above moments to enhance stability.
  - Our approach is Implicit LES.

Martin Geier, et. al., "Cascaded digital lattice Boltzmann automata for high Reynolds number flow"
Wall/Outer boundary

- Interpolated Bounce-Back (IBB) Scheme (1\textsuperscript{st} order)

\[ f_{-i}(x, t + \Delta t) = 2q f_i(x, t) + (1 - 2q) f_i(x - c_i \Delta t, t) - 2w_i \rho(x, t) \frac{e_i \cdot u_w}{c_s^2} \left( q_i < \frac{1}{2} \right) \]

\[ f_{-i}(x, t + \Delta t) = \frac{1}{2q} f_i(x, t) + \frac{2q - 1}{2q} f_{-i}(x, t) - \frac{1}{q} w_i \rho(x, t) \frac{e_i \cdot u_w}{c_s^2} \left( q_i \geq \frac{1}{2} \right) \]

where \( q_i = \frac{d_i}{r_i \Delta t} \), non-dimensional distance

※No wall function is used.


- Damping function is used for the outer boundary condition.

\[ f_{outer} = f - \alpha \left( f - f_{eq}^{target} \right) \]

\[ \alpha = 0.5 \times \left( \frac{d - r}{R - r} \right) \]

where \( r/R \) are inner/outer radius of damping region,
\( d \) is distance from inner radius \( r \)

Building-Cube Method

- BCM is a block-structured Cartesian grid approach proposed by prof. Nakahashi.

- Computational domain is divided into “Cubes”.

- Each cube has a uniform-spacing Cartesian grid, “Cells”.

- Cartesian grid & staircase representation → Simplification of grid generation & flow solver algorithm

- Equal spaced Cartesian grid in each cube → Higher-order spatial accuracy

- All cubes include same number of Cartesian grid → Easy handling of parallelization

- Change cube size locally → Easy adaptation of grids to local flow features
Domain Partition

- BCM framework uses both OpenMP/MPI parallelization.
- Z-ordering is used for MPI parallelization.

Grid information

<table>
<thead>
<tr>
<th>Details</th>
<th>2D</th>
<th>3D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimension</td>
<td>2D</td>
<td>3D</td>
</tr>
<tr>
<td>$Re$</td>
<td>$1.71 \times 10^6$</td>
<td></td>
</tr>
<tr>
<td>$M_{\infty}$</td>
<td>0.17</td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>5.5</td>
<td></td>
</tr>
<tr>
<td>Domain</td>
<td>$32L_{\infty} \times 32L_{\infty}$</td>
<td>$16L_{\infty} \times 16L_{\infty} \times 0.125L_{\infty}$</td>
</tr>
<tr>
<td>Cube</td>
<td>6535</td>
<td>77326</td>
</tr>
<tr>
<td>Cell</td>
<td>$32^2$</td>
<td>$4^3, 8^3, 16^3, 32^3$</td>
</tr>
<tr>
<td>Total cells</td>
<td>$6.7M$</td>
<td>$4.9M, 40M, 317M, 25B$</td>
</tr>
<tr>
<td>$\Delta x_{\text{min}}$</td>
<td>$1.22 \times 10^{-4}L_{\infty}$</td>
<td>$2.44 \times 10^{-4}L_{\infty}$</td>
</tr>
</tbody>
</table>

Periodic boundary condition is applied in spanwise direction.
2D results

- Flow separation is different at slat-cove compared to NS(RANS) results.
  ⇒ due to 2D computation with ILES.
- LBM overestimated $C_p$ compared to NS(RANS) results.

dp field@AoA=5.5

3D results
PSD data position

- B2: FaSTAR(L3)
- C1: FaSTAR(L2)
- D1: Present

PSD comparison

- B2: FaSTAR(L3)
- C1: FaSTAR(L2)
- D1: Present

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Takashi ISHIDA

**PSD comparison**

- **M7(α=5.5)**
  - B2: FaSTAR(L3)
  - C1: FaSTAR(L2)
  - D1: Present

- **F1(α=5.5)**
  - B2: FaSTAR(L3)
  - C1: FaSTAR(L2)
  - D1: Present

- **P1(α=5.5)**
  - B2: FaSTAR(L3)
  - C1: FaSTAR(L2)
  - D1: Present

- **P7(α=5.5)**
  - B2: FaSTAR(L3)
  - C1: FaSTAR(L2)
  - D1: Present

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**Time-averaged Cp**

- **30P30N(α=5.5)**

- LBM overestimated Cp at slat and flap ⇒ due to the outer domain size?
- BL thickness at slat-TE may be changed slightly compared to other CFD results due to the flow acceleration at flap suction.
Conclusions

- NBPs were well captured by present approach.

- The peak from slat-TE was slightly shifted to higher region compared to NS results, but reasonable agreement was obtained with experimental data.

- Future works
  - Grid convergence
  - Effect of local mesh refinement based on flow field
Thank you for your kind attention.