宇宙ごみのモデリングとその応用について
Orbital Debris Modeling and Applications

花田俊也（九州大学）
Toshiya Hanada (Kyushu Univ.)

宇宙ごみのモデリングにより、現在および将来のデブリ分布を予測する推移モデルを構築し、どのようにすればデブリを低減できるか、あるいは環境を改善できるか、を議論することができる。また、宇宙ごみのモデリングにより、地上光学望遠鏡を用いてどのように観測すれば効率的に未知のデブリを探索することができるか、並びに、地上から追跡できない小デブリの計測を軌道上で実施する際の実用性を評価できる。別の応用として、環境改善のために除去すべき人工物の姿勢運動を推定することもできる。この講演では、九州大学で注力している宇宙ごみのモデリングとその応用について紹介する。

The orbital debris modeling can build evolutionary models as essential tools to predict the current or future orbital debris populations, and also to discuss what and how to do for orbital debris mitigation and environmental remediation. The orbital debris modeling can also devise an effective search strategy applicable for breakup fragments in the geostationary region using ground-based optical sensors, and to evaluate the effectiveness of space-based measurements of objects not tracked from the ground, both to contribute to space situational awareness. Another application of the orbital debris modeling is to estimate attitude motion of space objects to be removed for environmental remediation. This paper briefly introduces efforts into orbital debris modeling and applications.
Orbital Debris Modeling

Orbital debris modeling mainly consists of debris generation and orbit propagation

- **Debris generation** can characterize and predict physical properties of fragments originating from explosions or collisions.
- **Orbit propagation** can characterize, track, and predict the behavior of individual or groups of space objects.

With collision flux estimation orbital debris modeling can build evolutionary models as essential tools to project the current or future orbital debris populations.

Orbital debris modeling is also useful and effective to improve the efficiency of measurements and to identify the location of breakups.
Debris Generation

A key element of modeling orbital debris environment is the ability to predict the outcome of a typical satellite fragmentation. There are two important factors for long-term orbital debris environment studies: (1) size, and (2) area-to-mass ratio distributions.

- **Size** distribution defines the number of fragments added to the environment after a breakup.
- **Area-to-mass ratio** distribution defines the orbital lifetimes of fragments with perigee altitudes < ~1000 km

**Shape** is important for improving the calculation of the average cross-sectional area of each fragments. **Shape** is also important for conducting a reliable assessment of the probability of non-penetration of spacecraft such as the Int’l Space Station.

Breakup Model

**Breakup Model**

- To characterize and predict physical properties of fragments originating from explosions or collisions

**Simulated spacecraft walls**

- To investigate low-velocity impacts on spacecraft
- The outcome was all non-catastrophic, resulting in craters or holes on simulated spacecraft walls

**CANSAT**

- To investigate the outcome of a catastrophic impact

**Micro satellites (under contract with NASA Orbital Debris Program Office)**

- To compare low-velocity and hypervelocity catastrophic impacts on identical micro satellites
- To investigate the effects of impact directions on fragmentation
- To investigate fragments originating from multi-layer insulation (MLI) and solar array panels (SAP)
Evolutionary Models

With collision flux estimation, orbital debris modeling can build **evolutionary models** as essential tools:

- To predict the current or future space debris environment, and also
- To discuss what and how to do for orbital debris mitigation and environmental remediation

**GEODEEM**
- To track objects in the geostationary region (or with eccentricity < 0.2, mean motion between 0.9 and 1.1 rev. per day, and inclination < 30 deg.)

**LEODEEM**
- To track objects in the low Earth orbit region (or with perigee altitude < 2000 km)

**NEODEEM**
- To track objects orbiting around the Earth
Solar Cycles Assumed after 2005

![Graph showing solar radio flux over time with different colors representing high, middle, and low solar activity.]

Black: high (75-230)
Red: middle (75-190)
Blue: low (75-160)

Impact of Solar Cycles

![Graphs showing space debris environment change in LEO with different models and atmospheric models.]
Measurements

Orbital debris modeling is also useful and effective to improve the efficiency of measurements.

Orbital debris modeling can characterize, track, and predict the behavior of groups of fragmentation debris to devise a practical method for ground-based optical measurements.

- Population prediction of fragments from a single breakup event specifies effectively when and how to conduct ground-based optical measurements.
- Motion prediction of fragments in a series of successive images clearly distinguishes between fragments originating from the target breakup event and the others.

This practical method has been verified by applying for two confirmed breakups in the geostationary region:

- Russian Ekran 2 (ID: 77092A) exploded on 23rd June 1978, and
Population Prediction and Observation Planning

Motion Prediction and Origin Identification

Motion Prediction

Origin Identification

Outcome of contract with JAXA

Detection rate = 0.5 [fragments/hour]
Detection rate = 14.7 [fragments/hour]

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Constraint Equations

Orbital debris modeling can characterize the nature of orbits on which debris impact with a space object.
- This characterization provides a constraint on a piece of debris which impacts with the space object
- This constraint can identify the location of on-orbit satellite fragmentations

Orbital debris modeling can also characterize the nature of orbits of fragments from a specific breakup event.
- This characterization can also provides a constraint on fragments from the specific breakup event, similar to the constraint above
- This constraint can identify the time of breakup events

Case Studies

Case A

Case B
Constraint on Orbital Plane

A constraint on the orbital plane of debris, which impact the measurement satellite, may be given by

\[ \mathbf{e}_W \cdot \mathbf{e}_R = \mathbf{e}_W \cdot \frac{\mathbf{r}_j}{r_j} = 0 \]

where

\[ \mathbf{e}_W = \begin{pmatrix} \sin \Omega(t) \sin i \\ - \cos \Omega(t) \sin i \\ \cos i \end{pmatrix} \]

Orbital Parameters to Estimate

Inclination: \( i \)

Right ascension of the ascending node at \( t = t_0 \): \( \Omega_0 \)

Nodal regression rate: \( \dot{\Omega} \)

Noted

\[ \Omega(t) = \Omega_0 + \dot{\Omega}(t - t_0) \]

and

\[ \dot{\Omega} \propto -\cos i \]
Constraints on $i$ and $\Omega_0$ Change with $\dot{\Omega}$ (Case A)

Two or More Measurement Satellites Needed
Constraints on $i$ and $\Omega_0$ Change with $\dot{\Omega}$ (Case B)

Constraint on Nodal Pression/Regression Rate

A constraint on the nodal precession/regression rate of the broken-up object may be given by

$$\begin{align*}
\mathbf{r}_1 \cdot \left[ \left( \mathbf{C}^3 \Omega(t_2-t_1) \right) \mathbf{r}_2 \right] \\
\times \left[ \left( \mathbf{C}^3 \Omega(t_3-t_1) \right) \mathbf{r}_3 \right] = 0
\end{align*}$$

where $[\mathbf{C}^3]$ represents a rotation matrix about the Earths’ axis of rotation with an angle given by the subscript.
Finding the Right Nodal Precession/Regression Rate (Case A)

\[
\begin{align*}
\dot{\Omega} & = \frac{v}{r_1 \cdot \left[ \left( \left[ \left( \frac{c_3}{\Omega(t_3)} \right) \right] \left[ \left( \frac{c_3}{\Omega(t_3)} \right) \right] \right] \right]} \\
& \times \left( \left[ \left( \frac{c_3}{\Omega(t_3)} \right) \right] \left[ \left( \frac{c_3}{\Omega(t_3)} \right) \right] \right) \\
& \times \left( \left[ \left( \frac{c_3}{\Omega(t_3)} \right) \right] \left[ \left( \frac{c_3}{\Omega(t_3)} \right) \right] \right)
\end{align*}
\]

\[\dot{\Omega} \text{ [deg/day]}\]

Finding the Right Nodal Precession/Regression Rate (Case B)

\[
\begin{align*}
\dot{\Omega} & = \frac{v}{r_1 \cdot \left[ \left( \left[ \left( \frac{c_3}{\Omega(t_3)} \right) \right] \left[ \left( \frac{c_3}{\Omega(t_3)} \right) \right] \right] \right]} \\
& \times \left( \left[ \left( \frac{c_3}{\Omega(t_3)} \right) \right] \left[ \left( \frac{c_3}{\Omega(t_3)} \right) \right] \right) \\
& \times \left( \left[ \left( \frac{c_3}{\Omega(t_3)} \right) \right] \left[ \left( \frac{c_3}{\Omega(t_3)} \right) \right] \right)
\end{align*}
\]

\[\dot{\Omega} \text{ [deg/day]}\]
Furumoto Constraint Equation

Furumoto derived a constraint equation that applies for the orbital parameters of a piece of debris impacted on a measurement satellite:

\[ R_{11} \sin i' \sin \Omega' - R_{21} \sin i' \cos \Omega' + R_{31} \cos i' = 0 \]

where

\[ R_{11} = \frac{x}{r} = \cos \Omega \cos u - \sin \Omega \sin u \cos i = \cos \delta \cos \alpha \]

\[ R_{21} = \frac{y}{r} = \sin \Omega \cos u + \cos \Omega \sin u \cos i = \cos \delta \sin \alpha \]

\[ R_{31} = \frac{z}{r} = \sin u \sin i = \sin \delta \]

Alternative Expressions for Furumoto Constraint Equation

Itaya derived alternative expression for Furumoto constraint equation:

\[ (R_{21}^2 + R_{31}^2)p^2 - 2R_{11}R_{21}pq + (R_{11}^2 + R_{31}^2)q^2 - R_{31}^2 = 0 \]

where

\[ p = \sin i' \cos \Omega' \text{ and } q = \sin i' \sin \Omega' \]

Hanada also derived alternative expression different from Itaya’s expression:

\[ R_{31}p^2 + R_{31}q^2 - 2R_{11}p + 2R_{21}q - R_{31} = 0 \]

where

\[ p = \tan \frac{i'}{2} \sin \Omega' \text{ and } q = \tan \frac{i'}{2} \cos \Omega' \]
Comparison between the Two Different Expressions

Itaya’s Expression

Hanada’s Expression

Summary

This paper briefly introduced efforts into orbital debris modeling and applications.

Orbital debris modelling can predict the stability of the current or future orbital debris populations to discuss what and how to do for the long-term sustainability of outer space activities.

Orbital debris modeling is also useful and effective to improve the efficiency of measurements and to identify the location/time of breakups.

Kyushu University is willing to pursue orbital debris modeling.
Microsatellite Impact Scenarios

Microsatellite Impact Fragmentation
Chinese Anti-satellite Test in Early 2007

Collision Flux of Fragments from Fengyun 1C along the Orbit of IDEA-1