Method of Particle Simulation for the Pulsar Magnetosphere by Use of Special Purpose Computer, GRAPE-6

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Abstract

The particle acceleration mechanism of pulsars has been an outstanding problem for more than forty years. A breakthrough can be provided by a large particle-simulation for the global magnetosphere showing the formation of the field-aligned electric field. However, this demands heavy calculation of Coulomb interaction. We have resolved this difficulty by use of GRAPE, which is a massively parallel special purpose computer for astronomical N-body problem at National Astronomical Observatory of Japan. Note that GRAPE can calculate Coulomb force as well as gravitational force because of the same inverse-square law and of sign-bit available with GRAPE. In this paper, we present a method of particle simulation for the global magnetosphere including electron-positron pair creation. We show some initial results of the simulation.

Key words: pulsars: general — magnetic field — plasmas

1. Introduction

The pulsed emission from the pulsars indicates that particles are accelerated to high energy up to 10^{12}eV. Although a variety of acceleration models were proposed, the problem of the particle acceleration is still outstanding.

A breakthrough can be given by a large particle-simulation for the global magnetosphere including the formation of the field-aligned electric field $E_{||}$, i.e., the simulation must not be by the usual MHD simulation. However, this sort of particle simulation demands calculation of Coulomb interaction which increases in proportion to the particle number squared.

2. GRAPE

We have succeeded in resolving this difficulty by use of GRAPE-6. GRAPE is a massively parallel special purpose computer for astronomical N-body simulations at National Astronomical Observatory of Japan. Typical use of GRAPE is for stellar dynamics. However, note that GRAPE can calculate Coulomb force as well as gravitational force because of the same inverse-square law and of sign-bit available with GRAPE. In case of gravitational force, the acceleration of i-th particle is calculated as

$$\boldsymbol{a}_i = \sum_{j\neq i}^{N} G m_j \frac{\boldsymbol{x}_j - \boldsymbol{x}_i}{(|\boldsymbol{x}_j - \boldsymbol{x}_i|^2 + \epsilon^2)^{3/2}}, \tag{1}$$

and in case of Coulomb force, the electric field at i-th particle is calculated as

$$\boldsymbol{E}_i = \sum_{j\neq i}^{N} q_j \frac{\boldsymbol{x}_j - \boldsymbol{x}_i}{(|\boldsymbol{x}_j - \boldsymbol{x}_i|^2 + \epsilon^2)^{3/2}}, \tag{2}$$

where N, G, and ϵ are total particle number, gravitational constant, and the softening parameter; m_j and q_j are mass and charge of j-th particle; $\boldsymbol{x}_i$ and $\boldsymbol{x}_j$ represent the position of i-th and j-th particle.

Since we look for a axisymmetric steady state, we ignore effects of time variation of the fields, and therefore the problem is electro-static and magneto-static. Thus the problem can be solved by GRAPE.

3. Model

In our simulation, we assumed as follows:

- A star is assumed to be an aligned rotator and a perfect conductor.
- The particles are emitted freely the stellar surface.

- The particles are subjected to the radiation drag force.
- Pair creation occurs in the region where the field-aligned electric field is stronger than a critical value.

Poisson equation

$$-\nabla^2\phi = 4\pi\rho, \tag{3}$$

is solved in use of GRAPE with the boundary conditions,

$$\phi(R_*) = \frac{\mu\Omega\sin^2\theta}{cR_*} + \text{const}, \;\; \phi(\infty) = 0, \tag{4}$$

where ϕ, ρ, R_*, μ, Ω, θ, and c are the electric potential, charge density, stellar radius, magnetic moment, angular velocity, colatitude, and speed of light, respectively.

For the particles, we use the equation of motion including radiation reaction force:

$$\frac{\mathrm{d}\boldsymbol{p}_i}{\mathrm{d}t} = q_i(\boldsymbol{E} + \boldsymbol{\beta}_i \times \boldsymbol{B}) + \boldsymbol{F}_{\mathrm{rad},i}, \tag{5}$$

where $\boldsymbol{\beta}_i$ is the velocity of the i-th particle in units of the light speed; $\boldsymbol{p}_i = \gamma_i m \boldsymbol{\beta}_i$, $\gamma_i = (1 - \boldsymbol{\beta}_i \cdot \boldsymbol{\beta}_i)^{-1/2}$; m and $q_i (= +q \text{ or } -q)$ are mass and charge of the particles, respectively; $\boldsymbol{F}_{\mathrm{rad},i} = (2/3)(q_i^2/R_c^2)\gamma_i^4(\boldsymbol{p}_i/|\boldsymbol{p}_i|)$ represents the radiation drag force. In contrast to the previous works, our simulation can treat cross-field drift motions due to the radiation drag and particle inertia.

4. Outline of Simulation

Our simulation proceeds in the following steps:

1. Start the calculation from the vacuum state.
2. Replace the surface charges by particles.
3. Solve the equation of motion of the particles.
4. Produce electron-positron pairs where needed.
5. Delete some particles.
6. Repeat the step 2 to 5 until the steady state is established.

Magnetospheric plasmas are represented by several tens of thousands of particles in the simulation. The surface charge, which increases in proportion to $E_{||}$, is replaced by particles because we assumed free emission. In the steady state, creation and loss of the particles balance to each other, so the total charge is determined automatically. Electron-positron pairs are generated in the region where $E_{||} > E_{\mathrm{crit}}$. We delete the particles which return to the star, go beyond the outer boundary, or are bound pairs.

5. Results

Figure 1 shows the particle distribution on the meridional plane. The flow consists of outflow and circulating component. The outflow component is considered as the pulsar wind.

Figure 2 is the strength of the field-aligned electric field on the meridional plane. It indicates "outer gap" that possesses $E_{||}$ in the small limited region. Electron-positron pair creation occurs in such regions.

6. Summary

In use of GRAPE-6, which is developed for the N-body problem, we performed a particle simulation for the global pulsar magnetoshere. The gravitational interaction is replaced by the Coulomb interaction. We show that this method is applicable for the pulsar problem. Although resolution is not so high, the simulation shows the outer gap and the cross-filed outflow due to radiation drag and particle inertia.

References

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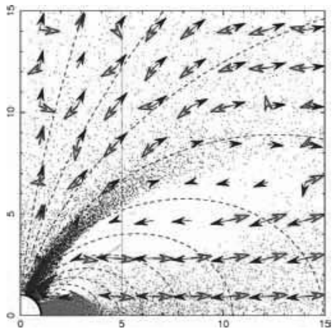

Fig. 1. The position of particles (dots) on the meridional plane. The filled and outlined arrows indicate the direction of the negative and positive flow. The geometrical scale length is normalized by the stellar radius, and the light cylinder corresponds to the axial distance of 5. The dashed lines represent dipole magnetic field.

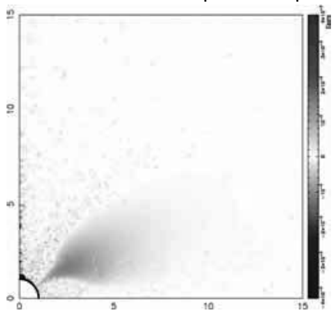

Fig. 2. The strength of the field-aligned electric field on the meridional plane.